

Physics-based Ankle Kinematics for Estimating Internal Parameters

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Abstract— This paper presents a kinematic model and its inverse solutions to identify internal ankle parameters in sagittal plane from external motion data; the effects of initial guesses and noise on the solutions are discussed. Experiments were designed to eliminate skin-marker errors in motion data, and performed to validate the model and the estimation method experimentally by comparing estimations with measurements from the computed tomography scan of the ankle. Based on observations that the skin temperature of bony prominences is lower than that of muscle, a novel infrared (markerless) method to locate bony prominences as features is conceptualized; experiments suggest that this concept feasibility is promising.

I. INTRODUCTION

The talocrural (ankle) joint plays a fundamental role in human locomotor system and takes part in the most of lower limb activities. To gain insights into the ankle joint for motion studies and medical applications, it is essential to develop physically intuitive models, which are verifiable with current medical imaging techniques, to determine characteristic ankle-parameters.

Research studies on the ankle/foot joint can be broadly divided into three major areas; bio-joint kinematics, visual representations of the joint interior and motion capture. Ankle/foot kinematics has been widely studied. From early models assuming a single degree-of-freedom (DOF) hinge joint and biaxial revolute joint [1-3] to more sophisticated mechanisms [4-5], the models gradually take into accounts the biological characteristics of a human joint with increased complexity. An online algorithm was proposed in [3] to estimate the joint-axis parameters, considering individual differences and axis-changes during motion. Based on experimental observations on the changes of ligaments during passive sagittal motion of an ankle joint, a 1-DOF model based on a 4-bar planar mechanism was proposed in [4] that was later extended to a spatial parallel mechanism [5]. These early models were empirical and developed with observations drawn from experiments on cadavers' ankles.

Recent advances in medical imaging techniques have enabled visual representations of the joint interior; among them, X-Ray Computed Tomography (X-CT) [6] and Nuclear Magnetic Resonance Imaging (MRI) [7] have been most commonly used to reconstruct the tomographic image or the internal structure of the joint of interests. X-CT has the advantages of high resolution and fast scanning but it has certain side effects [8]. MRI is a safe, fast and accurate

clinical diagnosis method, but this expensive imaging technique inevitably causes the subject certain discomfort [7]. Both these medical imaging techniques require the subject remain motionless during the scan.

For motion capture, most existing techniques employ optical systems other than inertial measurement systems [9], mechanical systems [10] and magnetic systems [11]. Optical measurement systems [12] use data captured from image sensors to triangulate the 3D position of an object among multiple cameras, where special markers (often coated with a retroreflective material) are generally required for data acquisition. The markers, attached on human skin or specially designed tight-fitting suits, may affect human movement; and advance preparation is needed in order to obtain high-quality data. Due to the elasticity of skin or suits, the markers may move relative to the underlying bone during the course of movement, leading to skin-marker errors [13-14]. Research on marker-less motion capture technology [15] is rapidly evolving. Current marker-free systems, which capture motion without wearing special equipment, have been limited by the complexity of unlabeled methods to obtain accurate 3D motion. A common solution is to estimate the parameters by a predefined human body model; for example, in [16] a method was presented to track human body by an extended Kalman filter after classifying 3D voxel of different parts. The method [16], however, cannot classify the body voxel data reliably and hence cannot obtain the correct body motion when the arms are close to the trunk or the feet are merged. Another method [17] uses algorithms to achieve human model acquisition and motion capture automatically, but it is unstable with large computation.

Despite extensive studies on ankle kinematics, there is a lack of methods to estimate the internal ankle-joint parameters to model the ankle kinematics, which are essential for design/control of compliant robotics consistent with human bio-motion [18]. Motivated by our early studies [19] indicating that bio-joint models are important for developing robots for mind, motor and motion recovery post stroke, this paper presents a new method along with an infrared method originally conceptualized in [20] to locate bony prominences to identify internal ankle parameters from motion data. The remainder of this paper are organized as follows:

- Section II begins with a physics-based model to describe the ankle motion in sagittal plane, providing a basis to solve a multivariable nonlinear regression problem for the internal parameters from measured features; initial guesses and noise effects on the solutions are discussed.
- Experiments were conducted to validate the kinematic models and investigate the effects of skin-markers on estimations in Section III; results are compared with measurements from the CT-scan of the ankle.
- A novel infrared method directly locates bony prominences as features to eliminate skin-marker errors is proposed. Specific findings are concluded in Section IV.

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II. TALOCRURAL (ANKLE) JOINT KINEMATICS

Figure 1 shows the notations and coordinate systems used in modeling the motion of a typical talocrural joint, where the deformation of the interosseous ligament between the tibia and fibula is assumed negligibly small. The shank (tibia/fibula) is treated as a rigid body and characterized by a plane defined by three bony prominences denoted as red filled dots in the lateral and posterior views in Fig. 1(a, b); namely, the head (HF) and lateral malleolus (LM) of the fibula and the tibia medial malleolus (MM). For describing the plantarflexion (PF) and dorsiflexion (DF) motions of an unloaded ankle as an open-chain mechanism, the talus with the foot are modeled as a rigid body characterized by three identifiable “landmarks” denoted as blue open-circles at the upper ridge of the calcaneus (RC) and the dorsal aspect of first and fifth metatarsal heads, (FM and VM). For design/control of lower extremity exoskeletons, it is desired to develop a model to determine the characteristic parameters of the ankle joint and the relative motion between the two planes; LM-HF-MM and RC-FM-VM.

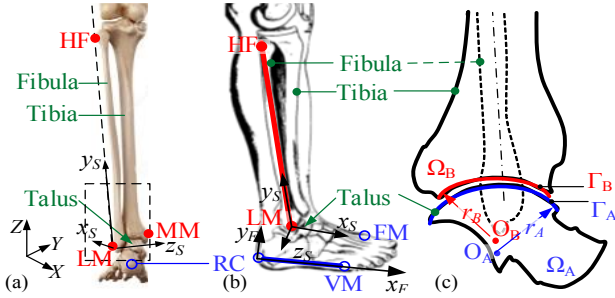


Fig. 1 Illustration of an ankle joint. (a) Posterior view, left leg. (b) Lateral view, right leg. (c) Characteristic parameters, left leg.

The ankle joint is often assumed one-DOF permitting rotation along a predetermined path, where the rotational axis lies between the tips of the malleoli and is offset in two anatomic (frontal and transverse) planes. In Fig. 1(b), the local coordinate system $x_s y_s z_s$ (based on right hand rule) is assigned at LM with its y_s -axis pointing toward HF and x_s -axis perpendicular to the plane (LM-HF-MM). Similarly, a local frame $x_f y_f z_f$ is assigned at RC with its x_f -axis pointing toward VM and y_f -axis directing along the normal to the plane (RC-FM-VM). The position vectors of (LM, HF, MM) and (RC, FM, VM) are assumed known with respect to a reference frame XYZ . The unit vectors ($\hat{X}, \hat{Y}, \hat{Z}$) of these coordinate systems are defined by (1a, b) where $\mathbf{P}_s = \mathbf{P} - \mathbf{P}_0$:

$$\begin{bmatrix} \hat{X}_F & \hat{Y}_F & \hat{Z}_F \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{VR} & \mathbf{P}_{VR} \times \mathbf{P}_{FR} \\ |\mathbf{P}_{VR}| & |\mathbf{P}_{VR} \times \mathbf{P}_{FR}| \end{bmatrix} \quad (1a)$$

$$\begin{bmatrix} \hat{X}_s & \hat{Y}_s & \hat{Z}_s \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{ML} \times \mathbf{P}_{HL} & \mathbf{P}_{HL} \\ |\mathbf{P}_{ML} \times \mathbf{P}_{HL}| & |\mathbf{P}_{HL}| \end{bmatrix} \quad (1b)$$

The commonly used (1a, b) are black box in nature. Unlike a pin-joint allowing free rotation about a fixed axis, contact surfaces of a muscle-actuated ankle are connected by ligaments and thus characterized by an infinite number of rotation axes but with very limited motion range (of rotation angle and center). To gain insights into the bio-joint for design and control of an exoskeleton, a physics-based kinematic model is derived to identify the internal parameters of an ankle joint from motion data.

A. Physics-based Kinematic Model

The talocrural joint is a mortise joint rotating about a nominal ankle-axis with a relatively small range ($\approx \pm 8^\circ$) of inclination between the joint axis and the midline of tibia in the front plane. For describing the motion of the talus Ω_A relative to the shank Ω_B (Fig. 1c), the contact surfaces (Γ_i where i denotes A or B) are assumed circular (radius r_i , center O_i) in the xy plane. In Fig. 2(a), θ is the rotation of Ω_A about O_A ; and ϕ is the angular distance between the (present and next) “common” points (C and C’) about O_B . The term “contact point” is avoided here because the ankle has a relatively large load-bearing surface area lowering the stresses over the joint; instead, the term “common point” along with the parameter ϕ_0 is used loosely to reduce complex bio-lubrication mathematically to a trackable form while accounting its kinematic effect. The talus Γ_A rolls and slides relative to the shank Γ_B when the foot flexes. Thus, the problem reduces to determine the motion of Ω_A (characterized by center O_A and vector $\mathbf{N}$) relative to Ω_B (center O_B) in terms of the following parameters (Fig. 2a):

- ϕ_0 representing the initial “common” point measured from a predefined position (or reference tibia position).
- $r_B - r_A$ or the difference between two radii of the circular surfaces (Γ_A and Γ_B).
- η , referred to here as “sliding factor” accounting for the slipping between Γ_A and Γ_B , that has 3 special cases:

$$\eta = \frac{\phi}{\theta} = \begin{cases} 0 & \text{Rotation about a fixed axis, } \phi = 0; \\ \infty & \text{Pure sliding with no rotation, } \theta = 0; \\ r_a / r_b & \text{Non-slip rolling on a circular arc.} \end{cases}$$

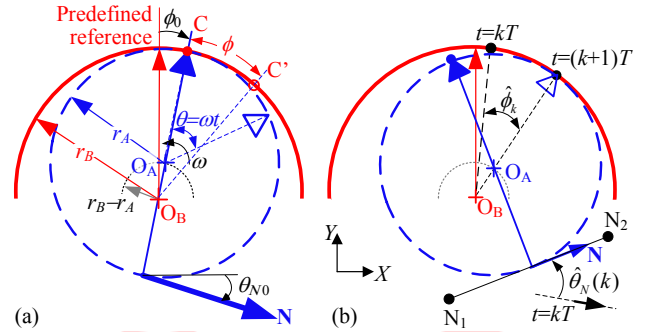


Fig. 2 2D ankle-joint motion in sagittal plane. (a) Characteristic parameters. (b) Kinematics between k^{th} and $(k+1)^{\text{th}}$ instants.

To determine the unknown parameters, the kinematics is formulated in discrete-time representation with parameters defined in Fig. 2(b) where the position vectors at the k^{th} and $(k+1)^{\text{th}}$ time instants are related by the rotational matrix $[\mathbf{R}]$:

$$\mathbf{X}(k+1) = [\mathbf{R}(k+1)] \mathbf{x}(k) + \mathbf{O}_i(k+1) \quad (2a)$$

where $kT \leq \Delta t < (k+1)T$ and $i=A, B$.

$$\mathbf{x}(k) = \mathbf{X}(k) - \mathbf{O}_i(k). \quad (2b)$$

$$\eta = \begin{cases} 0 & \mathbf{R} = \mathbf{R}_1(\hat{\theta}_k) \text{ Rotate about a fixed axis} \\ \infty & \mathbf{R} = \mathbf{R}_2(-\hat{\phi}_k) \text{ Sliding with no rolling} \\ (0, \infty) & \mathbf{R} = \mathbf{R}_N(\hat{\theta}_N) \text{ Roll and slide} \end{cases} \quad (3)$$

The specific form of $[\mathbf{R}(k)]$ in (2a) depends on the type of

relative motion between Ω_A and Ω_B , which can be characterized by sliding factor η in (3), where

$$\mathbf{R}_N(k) = \mathbf{R}_1 \mathbf{R}_2 = \mathbf{R}(\hat{\theta}_N) = \begin{bmatrix} \cos \hat{\theta}_N(k) & -\sin \hat{\theta}_N(k) \\ \sin \hat{\theta}_N(k) & \cos \hat{\theta}_N(k) \end{bmatrix} \quad (4)$$

$$\hat{\theta}_N(k) = \hat{\theta}_k - \hat{\phi}_k = \arg \mathbf{N}(k) - \arg \mathbf{N}(k-1) \quad (5a)$$

$$\theta_N(k) = \theta_{N0} + \sum_{i=1}^k \hat{\theta}_N(i) \quad (5b)$$

$\eta_k = \hat{\phi}_k / \hat{\theta}_k$; and $(\hat{\theta}_k, \hat{\phi}_k)$ are the incremental values between the $(k-1)^{\text{th}}$ and k^{th} instants.

As an illustration, consider Fig. 2(b) where Ω_A rolls relative to Ω_B with $\eta = (0, \infty)$ and $m=2$ (N_1 and N_2) such that $\hat{\theta}_N(k)$ defined in (5) can be measured from $\mathbf{N}$ that points from N_1 to N_2 . The forward kinematics that determines the Ω_A motion relative to the (fixed) Ω_B at the k^{th} time instant is given in (6a, b):

$$\mathbf{X}_{N_1}(k) = \mathbf{R}_N(k) [\mathbf{X}_{N_1}(k-1) - \mathbf{O}_A(k-1)] + \mathbf{O}_A(k) \quad (6a)$$

$$\mathbf{O}_A(k) = \mathbf{R}_2(k) [\mathbf{O}_A(k-1) - \mathbf{O}_B] + \mathbf{O}_B \quad (6b)$$

$$\text{and } \mathbf{O}_A(k) = (r_B - r_A) [\sin \phi_k \quad \cos \phi_k]^T + \mathbf{O}_B \quad (6c)$$

B. Internal Parameter Identification

For the bio-joint problem described above, the external features can be located from measurements, but the internal joint parameters are unknown. An *inverse* kinematics is formulated by rewriting (6a) to 7(a, b) to solve for the unknown parameters from a time sequence of n images, each of which includes m measurable features:

$$-\mathbf{R}_N(k) \mathbf{O}_A(k-1) + \mathbf{O}_A(k) = \mathbf{M}(k) \quad (7a)$$

$$\text{where } \mathbf{M}(k) = \mathbf{X}_{N_1}(k) - \mathbf{R}_N(k) \mathbf{X}_{N_1}(k-1) \quad (7b)$$

In (7a), $\mathbf{M}(k)$ can be calculated from measurements. Assuming that $(r_B - r_A)$ and η are approximately constant, the unknown $\mathbf{O}_A$ in the left side of 7(a) can be expressed relative to $\mathbf{O}_B$ using (6c), which leads to (8a, b):

$$\mathbf{F}_k(r_B - r_A, \eta, \phi_0, X_B, Y_B) = \mathbf{Q}(k) + [\mathbf{I} - \mathbf{R}_N(k)] \mathbf{O}_B - \mathbf{M}(k) = 0 \quad (8a)$$

$$\frac{\mathbf{Q}(k)}{2(r_B - r_A)} = \begin{bmatrix} \sin \frac{\phi_k + (\hat{\theta}_N(k) - \phi_{k-1})}{2} \cos \frac{\phi_k - (\hat{\theta}_N(k) - \phi_{k-1})}{2} \\ -\sin \frac{\phi_k + (\hat{\theta}_N(k) - \phi_{k-1})}{2} \sin \frac{\phi_k - (\hat{\theta}_N(k) - \phi_{k-1})}{2} \end{bmatrix} \quad (8b)$$

$$\text{and } \phi_k = \phi_0 + \sum_{i=1}^k \hat{\phi}_k = \phi_0 + \frac{\eta}{1-\eta} \sum_{i=1}^k \hat{\theta}_N(i) \quad (8c)$$

From (8a~c), an inverse problem is formulated to solve the 5 unknowns $(r_B - r_A, \eta, \phi_0, \mathbf{O}_B[X_B, Y_B])$, which consists of two nonlinear scalar (x and y) equations. With $(k = 1, \dots, n)$ images, an objective function (9) consisting of $2n$ scalar non-linear equations can be derived from (8a~c) to formulate a multivariable nonlinear regression problem to solve for the five unknowns by minimizing $\mathbf{F}$,

$$\mathbf{F}(r_B - r_A, \eta, \phi_0, X_B, Y_B) = \mathbf{0} \quad (9)$$

To provide a basis to evaluate the effect of initial guesses on the solutions, a residual norm J is defined in (10) where $\hat{\mathbf{F}}$ is the function $\mathbf{F}$ evaluated with the estimated solutions:

$$J(r_B - r_A, \eta, \phi_0, X_B, Y_B) = \sum_{i=1}^m \|(\hat{\mathbf{F}} - \mathbf{F})\|^2 \quad (10)$$

A residual norm $J=0$ indicates that the specific set of guesses lead to correct solutions. Once joint internal parameters are estimated, $\mathbf{O}_A$ and hence the general motion of the talus Ω_A relative to the tibia/fibula Ω_B can be determined from the forward kinematics given in (6a~c).

C. Effects of Initial Guesses on Inverse Solutions

The nonlinear regression problem can be numerically solved in MATLAB using the *lsqnonlin* command, but it requires a set of initial guesses. Without loss of generality, unity parameters are used in the numerical simulations. Numerical trials suggest that the solutions to the nonlinear equation (9) derived from (8a) are tolerant to a relatively wide range of initial $(\phi_0, \mathbf{O}_B)$ guesses but highly sensitive to the two other unknowns $(r_B - r_A)$ and η . Since the parametric values for a human are within a certain range, feasible guesses can be pre-calculated and stored as a reference table.

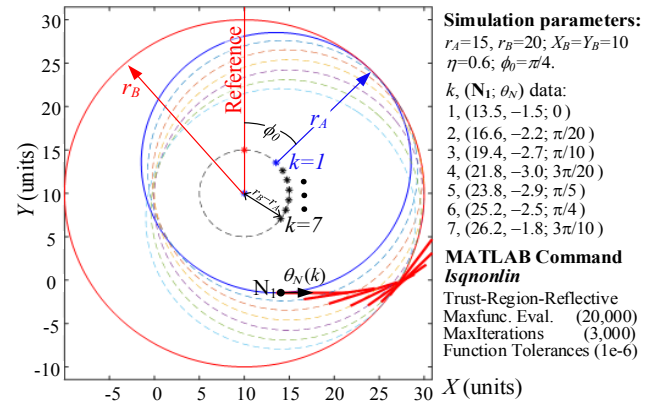


Fig. 3 Simulated forward kinematics.

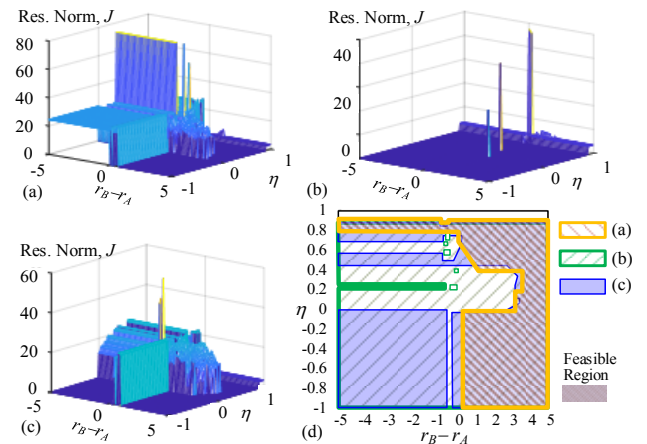


Fig. 4 Effect of initial guesses on residual norm for three $(\phi_0, \mathbf{O}_B)$. (a) (0, 0, 0). (b) (0, 5, 5). (c) $(\pi/6, 0, 0)$. (d) Feasible region.

The procedure is best illustrated with a numerical example shown in Fig. 3, where a discrete set of seven data, N_1 and $\theta_N(k)$ where $k=1, 2, \dots, 7$) simulating the forward kinematics (6a~c) of an ankle is displayed along with the listing of the parametric values used in the computation. The MATLAB command *lsqnonlin* (with options listed on the

lower right of Fig. 3) was used to solve the nonlinear equation (9). To help visualize, Figs. 4(a~c) graph the residual norm J computed with 3 different sets of initial $(\phi_0, \mathbf{O}_B)$ guesses over the ranges of the other two unknowns:

$$-5 \leq (r_B - r_A) \leq 5 \text{ and } -1 < \eta < 1$$

In Fig. 4(d), the $J=0$ regions for three different initial $(\phi_0, \mathbf{O}_B)$ guesses are superimposed illustrating the feasible regions of initial guesses for the simulated data.

The noise effect on the estimated solutions was simulated by adding (10% and 30%) random noise to the seven (N_1, θ_N) data; and their residue norm J values of the least-square optimization are compared with the noise-free solutions in Table I. The noise-induced errors in $(r_B - r_A, \eta, \phi_0, X_B, Y_B)$ are (18%, 1.7%, 7.5%, 16%, 3.5%), suggesting that $(r_B - r_A)$ and X_B are most sensitive to noise while η is least sensitive. In all cases, closely zero J can be obtained; and for a 30% random noise, maximum errors are within 20%.

Table I. Effects of random noise on estimation (units)

% [rand]	$r_B - r_A$	η	ϕ_0	$\mathbf{O}_B (X_B, Y_B)$	J
0	5.00	0.6	0.79	(10.00, 10.00)	0.00
10	4.16	0.63	0.65	(11.46, 9.85)	0.04
30	5.90	0.59	0.85	(8.40, 10.35)	0.28

III. RESULTS AND DISCUSSION

As the success of the method depends on reliable measurements of the external features relative to the ankle-joint, problems commonly encountered in existing feature-extraction methods were investigated. Based upon the findings, experiments were designed to validate the physics-based kinematic model (Section II), and evaluate its effectiveness for identifying the internal parameters of an ankle-joint from external-feature measurements. A novel method based on infrared imaging, which directly locate the bony-prominences as features, is then proposed.

A. Design of External Features and Experimental Setup

For imaging motion data, two retroreflective markers (P_1 and P_2) are assigned at the bony prominences (HF, LM) of the shank, and two others (N_1, N_2) at (VM, RC) of the foot; N_2 is placed along the line from VM to RC for 2D imaging in the sagittal plane. However, human muscles are deformable causing skin movements relative to the bony targets. As a result, the markers that appear in the images may not be the actual location of the bony targets, which are referred to here as *skin-marker errors*.

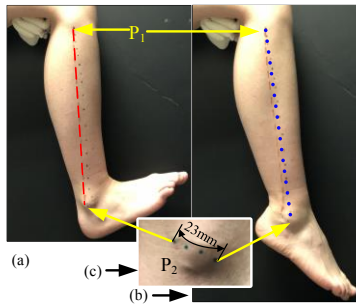


Fig. 5 Skin-marker errors. (a) Full dorsiflexion. (b) Full plantarflexion. (c) Skin movements between DF and PF.

To analyze the effect of the skin-markers on the planar motion measurement of an unloaded lower-leg, the (HF, LM) bony prominences were manually located; red-dash line at full DF (Fig. 5a) and blue-dot line at full PF (Fig. 5b). Very

small skin movement was observed around the HF, VM and RC. However, the skin-marker around the LM deviates significantly (green dots in Fig. 5c) because of the soleus, peroneus brevis and tibialis anterior muscle flexing. For verifying the ankle kinematics, the unloaded foot (N_1, N_2) motion is described experimentally relative to the *fixed* shank characterized by HF (marker P_1) and a 2nd point. Two methods to select the 2nd point: 1) The initial P_2 location, and 2) a relatively stationary point P between P_1 and P_2 .

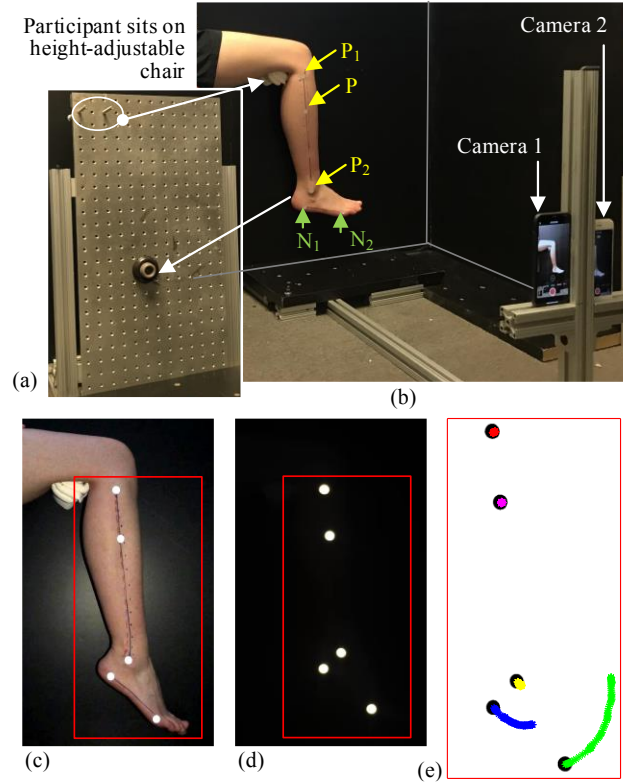


Fig. 6 Experiments (a) Aluminum frame supporting the lower leg. 2) Overall experimental setup. (c) Regular image at $k=1$. (d) Retroreflective image at $k=1$. (e) Marker motion data ($n=82$).

As shown in the experiment setup in Figs. 6(a, b), the upper leg is supported; the MM is positioned but allowed to rotate freely on the vertical Aluminum (Al) frame such that the shank and hence (HF, LM) remain stationary as the foot was gradually flexed between full DF to full PF. A non-reflective black board was placed between the Al frame and the subject's leg to facilitate imaging by a pair of iPhones (1080 HD, 30fps) which were placed parallel to the sagittal plane to capture n sequential video-images. Figs. 6(c, d) show a typical pair of (regular, retroreflective) images at $k=1$. The discrete-time trajectory data (Fig. 6e) of the markers, which characterize a motion cycle of the ankle (DF and PF), are computed from $n=82$ sequential images using image processing tools for subsequent analyses. As expected, the (P_1, P) data computed from the 82 frames are nearly identical while same noticeable P_2 variations can be observed.

The parameters $(r_B - r_A, \eta, \phi_0, \mathbf{O}_B)$ of a participant (female, age 22) were estimated from (9) using measured (N_1, N_2) with respect to the shank. The solutions were computed using three different (shank-locating) methods to investigate the effects of features on the estimations,

Method A: With (HF, LM) fixed, only the initial (P_1, P_2) pair at $k=1$ was used as a coordinate-reference with (N_1, N_2) .

Method B: As in the conventional methods, the pair (P_1, P_2) was used as instantaneous references with (N_1, N_2).

Method C: As in Method B but P_2 is replaced with P where the skin is less sensitive to the muscles that flex the foot. The solutions are compared in Table II and their validations are discussed in Section III.B.

Table II. Effects of features on estimated solutions

Methods	$r_B - r_A$ (mm)	η	ϕ_0 (rad)	$O_B = [X_B, Y_B]$ (pixel)
A. HF, LM	0.733	0.85	3.86	[213.1, 685.8]
B. P_1, P_2	0.082	0.05	0.00	[197.5, 686.6]
C. P_1, P	1.730	0.83	1.80	[216.3, 669.5]

B. Experiment Validation of Physics-based Kinematics

A CT-scan (Fig. 7) of the participant's ankle (Fig. 6) at plantarflexion was performed at the University Hospital of Huazhong University of Science and Technology for model validation. Typical sectional and isometric views of the CT-scan are displayed (using Materialise Mimics Innovation Suite Research 19.0) in Fig. 7. Since η and ϕ_0 cannot be determined from the static CT scan, the radial difference ($r_B - r_A$) and the distance between O_B and the projected LM on the sagittal plane were measured at nine sections (Fig. 7, top-left). For each CT section, the radii (r_A, r_B) were measured by circle-fitting of three points on the (Γ_A and Γ_B) contours, from which the average values of ($r_B - r_A$) and projected LM- O_B distance from the nine measurements were compared with the solutions in Table II.

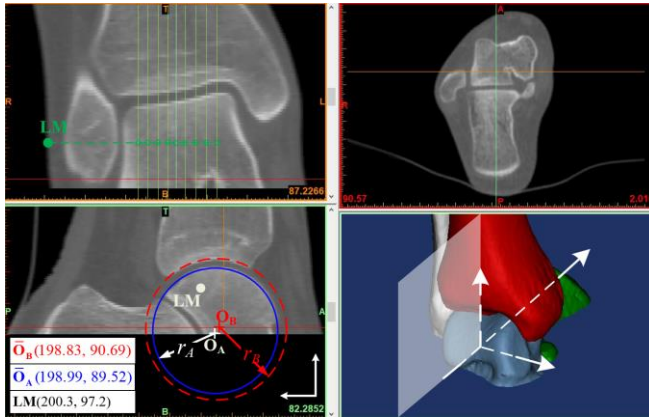


Fig. 7 CT sectional/isometric views of the ankle in Figs. 5 and 6.

To help visualize the physics-based kinematics (6a~c), ($r_B - r_A, \eta, \phi_0, O_B$) are illustrated graphically in Fig. 8(a) where $O_A(k)$ and $\theta_A(k)$ are solved for a specified ϕ_k ; and N at $\theta_N \approx 0$ corresponds to the 41th image frame. Since only ($r_B - r_A$) is solved, the average r_B value from the CT is used for graphic presentation in Fig. 8(a). The inverse solutions (Table II) are quantitatively compared with that obtained from the CT in Figs. 8(b, c) where measurements of ($r_B - r_A$) (black dots) and projected O_B -LM distance, and their average values (thin dashed lines) are plotted.

The following are observations drawn from the results:

- As shown in Figs. 8(b, c), the results estimated by Method A are within the measurement ranges. The ($r_B - r_A$) data from the CT have large variations ranging from 0.2 to 1.85mm; the difference between the estimated and CT average (1.07mm) is within 1.5% of average r_B (22.7mm). The estimated projected O_B -LM distance is about 9% from the average (7.37mm).

- Despite the large CT variations, the results estimated by Methods (B and C) relying on skin markers are outside the ranges of the CT-measured data suggesting that the skin-marker errors may not be neglected.
- Although the specific ϕ_0 value cannot be identified from the CT, the solution can be justified by observing the position of O_A relative to O_B . Experimentally, the motion started at full plantarflexion where the CT-scan was taken at the same pose. As illustrated in Fig. 8(a), the initial position of O_A ($k=1$) estimated by ϕ_0 is at the lower left of O_B , which is closely similar to that in the CT, suggesting that the solutions are reasonable.

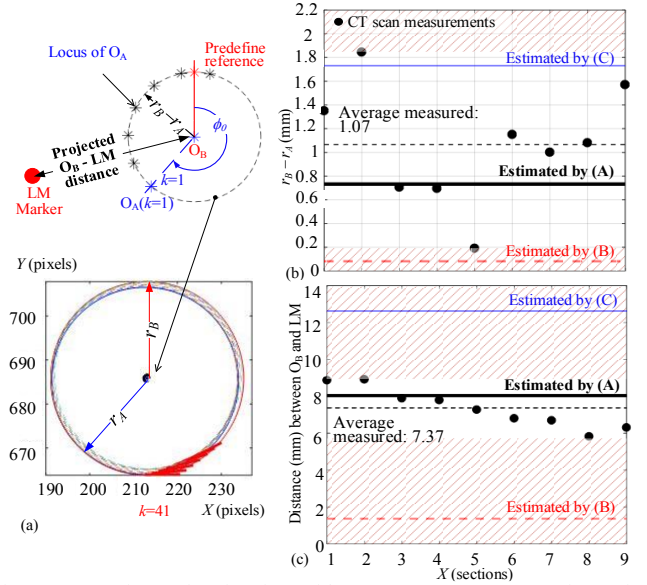


Fig. 8 Comparison of estimation with CT measurement. (a) Kinematic simulation. (b) $r_B - r_A$. (c) Projected O_B -LM distance.

C. Infrared-based Direct Bony-prominence Locating

The skin temperature of bony prominences is lower than that of the muscle as the latter produces heat. Based on this observation, a novel infrared method to locate bony prominences is conceptualized, which avoids skin-marker errors commonly encountered in conventional methods. The method is illustrated in Fig. 9 where the experiments were conducted on a setup similar to Fig. 6(b), but the MM (of a 2nd female subject, age 22) was not fixed. The (infrared and visible-light) images were captured experimentally by an 8-bit FLUKE Ti400 infrared thermal camera (320×240 pixels, 9 f/s). For concept validation, the actual (HF, LM) are manually marked on the leg before imaging (Fig. 9a). Fig. 9(b) shows the corresponding infrared image, from which the isotherms/streamlines are plotted in Fig. 9(c). As illustrated with the zoom-in region (40×40 pixels) where the temperature reaches static state in Figs. 9(d, e), the (HF, IM) are located by searching the local minimum temperature in their neighborhoods where the streamlines (normal to the isotherms) converge along the opposite gradient direction.

Results evaluating the concept are summarized in Fig. 10, from which the following findings can be drawn. As shown in Figs. 10(a~c) for the images at (PF, DF), the local minimum temperature at LM is similar to that at HF. In this experiment, the minimum temperatures at (HF, LM) were about 27°C; and the temperature gradients around the bone prominences were about 0.085°C/pixel. As the (HF-LM)

distance is fixed, ($D_i, i=1, 2, 3$) should yield the same result. Table III (Fig. 9, bottom right) tabulates the %Diff defined as $100(IR-D_A)/D_A$ where the physically measured (HF-LM) distance is $D_A(=352\text{mm})$, showing the errors are within 6%.

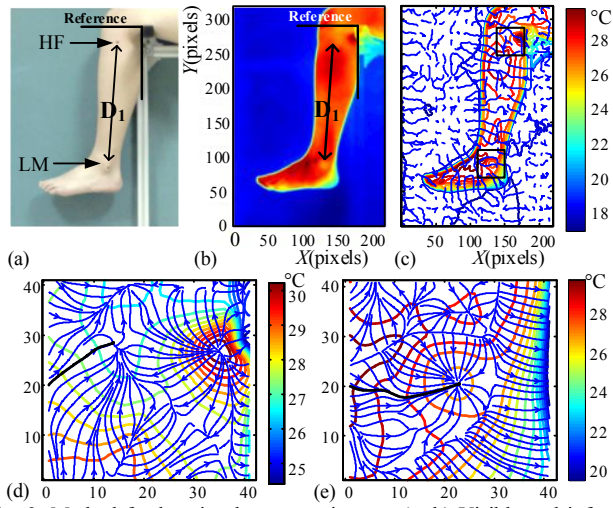


Fig. 9. Method for locating bony prominences (a, b) Visible and infrared images (c) Isotherms/streamlines. (d, e) Zoom-in (HF, LM).

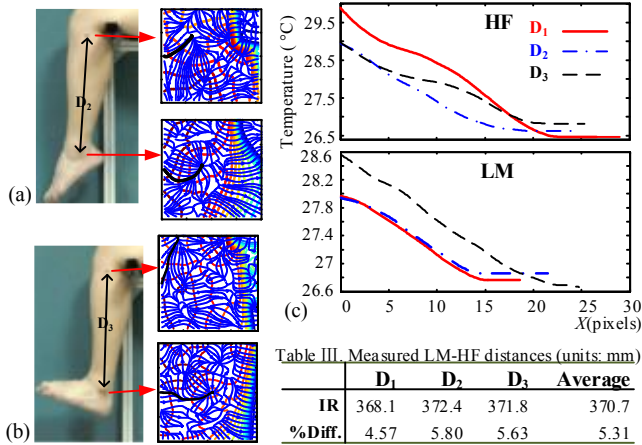


Fig. 10 Results (a, b) HF and LM at Full PF and at Full DF. (c) Streamline temperature in zoom-in regions.

IV. CONCLUSION

A physics-based kinematic model and its inverse model have been derived to identify five internal parameters of an ankle joint from motion data; (r_B-r_A) , η , ϕ_0 , and $\mathbf{O}_B$. Numerical investigations reveal that the solutions are slightly influenced by the initial $(\phi_0, \mathbf{O}_B)$ guesses and identify the regions of feasible initial (r_B-r_A, η) guesses, and relatively robust to noise. Using experiments designed to eliminate the skin-marker errors, the ankle kinematics and its inverse solutions have been validated by comparing estimations relative to the CT measured average: the estimated (r_B-r_A) error is within 1.5% of the average r_B ; and the error of the estimated projected $\mathbf{O}_B$ -LM distance is about 9%. The estimated ϕ_0 indicates that the position of $\mathbf{O}_A$ relative to $\mathbf{O}_B$ is closely similar to that observed in the CT, suggesting that the solutions are reasonable. The parameter η cannot be determined from the static CT-scan; the kinematics provides an effective means to overcome this limitation. Experimental findings suggest that the skin- marker error at LM may not be neglected even if the shank is position-fixed. The interests to

locate bony-prominences directly as features have led to the conceptualization of a novel temperature-based method, which have been validated its feasibility to locate bony prominences and eliminate skin-marker errors.

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