

# Machine Perception Based on Field Reconstruction for Conductivity and Hidden Geometrical Feature Characterization

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**Abstract**— This article presents a new machine-perception method based on eddy current (EC) effects to reconstruct the electrical-conductivity field of a nonferrous material and reveal hidden geometrical features commonly encountered in intelligent manufacturing using a one-time finite magnetic flux density (MFD) measurements. By discretizing the conductor, the measurement models of the physical fields (based on discrete MFD measurements) are linearly established. The reconstruction of a physical field is formulated as a linear inverse problem and solved using Tikhonov regularization method. The EC-based machine-perception is illustrated by reconstructing the conductivity field and hidden geometrical feature of aluminum plates. Additionally, the effects of different sensor configurations on the reconstruction results are analyzed numerically. Using an experimental prototype consisting of an air-cored electromagnet and a two-dimensional array of magnetic sensors with associated electronics, the effectiveness of the machine-perception method and the accuracy of the reconstructed physical field are demonstrated numerically and experimentally.

## I. INTRODUCTION

Smart sensing and machine-perception are attracting more and more attention in many research fields, and are in exponentially growing demand for intelligent manufacturing in the last two decades. High-quality manufacturing with superior service capability has been challenging due to special requirements and uncertain environments, such as workpiece deformation and vibration under uncertain cutting forces and thermal effects when machining thin-walled components (for examples, compressor disks and casings in aircraft engines) [1] [2]. The effectiveness of the real-time parameter-sensing and process-monitoring dictates the success of manufacturing. For the metallic-additive-manufacturing, most of the critical problems focus on process reliability of the finished-part geometries and material properties, which require layer by layer defect detection as well as post-processing precision machining [3] [4]. For such applications with defect detection between layers, the real-time estimation of the geometry and material properties are needed to be performed within large areas or volumes.

Eddy current induced inside the electrical conductor due to the presence of a changing magnetic field has many attractive characters (such as great penetration, fast response, well

defined theory, and insensitivity to oil or other media), which are well suitable for nondestructive sensing and testing in intelligent manufacturing [5] [6] [7]. For instance, the estimation of electrical conductivity, which has the potential applications in nondestructive evaluation of multiple material properties (such as subsurface residual stresses [8] [9]), is normally performed using eddy-current technologies. Based on the electromagnetic coil excitation, there are two primary types of EC-based methods for conductivity estimation: 1) frequency-domain analysis using sinusoidal excitation [10] [11], and 2) time-domain analysis using pulsing transient excitation [12] [13]. Another good application is the detection of internal defects or non-conductive buried materials (such as embedded sensors in a 3D-printed product) which can be interpreted as hidden geometrical features. Different from the conductivity estimation, most of the methods for defect detection are based on time-domain analysis using pulsing transient excitation [14] [15] [16]. By relating the transient specifications (such as rise-time, peak-time) to the corresponding patterns of surface- and/or subsurface-defects, defects can be classified.

However, even though the EC technology has been widely used, most traditional EC testing methods estimate or measure the target parameters at discrete points or within small areas/volumes. For the applications requiring detection and measurement within large areas or volumes, such as metallic additive manufacturing, the sensor probe needs to scan the whole areas, which limits the scopes of the applications and dramatically affect manufacturing efficiency [17] [18]. Meanwhile, many traditional eddy-current testing methods (such as defect detection) are based on pre-knowledge or pre-experimental result used to establish the relationship and perform calibration, which is also time consuming and may not be practical for certain applications.

To overcome the above-mentioned problems, a new sensing technique based on physical field reconstruction has been developed for intelligent manufacturing, which has potentials to integrate a number of multi-functions in a compact sensor for estimating or detecting desired parameters within a large area/volume using one-time measurement.

- 1) The measurement models of physical fields based on discrete MFD measurements are linearly established. The model for continuous field reconstruction (such as conductivity) is based on linearization whereas the model for geometrical feature detection is based on the properties of superposition. The physical field reconstruction is transformed to linear inverse problem, which is solved using Tikhonov regularization method.
- 2) The effectiveness and accuracy of the reconstruction method are demonstrated numerically. The conductivity

This work was supported in part by the U. S. National Science Foundation EFRI-M3C-1137172, CMMI-1662700, and National Basic Research Program of China (973 Program, Grant No. 2013CB035803).

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field and hidden geometrical feature of aluminum plates are estimated. Meanwhile, the effects of sensor configuration on the reconstruction results are analyzed numerically. The effects of different sensor configurations characterized by the sensor interval on reconstruction are analyzed.

- 3) An experimental setup consisting of a prototype sensor array, an air-cored electromagnet (EM) and associated hardware is utilized to successfully reconstruct the physical (conductivity and hidden geometrical feature) fields.

## II. PHYSICAL FIELD RECONSTRUCTION

The schematics in Fig. 1 illustrates the physical field-based machine-perception method for characterizing the electrical conductivity and hidden geometrical features within the (non-ferrous, anisotropic) material in a time-varying magnetic field. As shown in Fig. 1, the method, which uses a two-dimensional array of sensors to measure the MFD generated by the induced eddy-current density (ECD) field in the workpiece (WP), involves two steps: The first step formulates a measurement (forward) model to analytically relate the measured ECD-generated MFD to the physical properties of the WP. Since the ECD depends on the physical properties of the conductor, the second step reconstructs the physical fields from the finite measurements by solving the mathematically linear inverse problem.

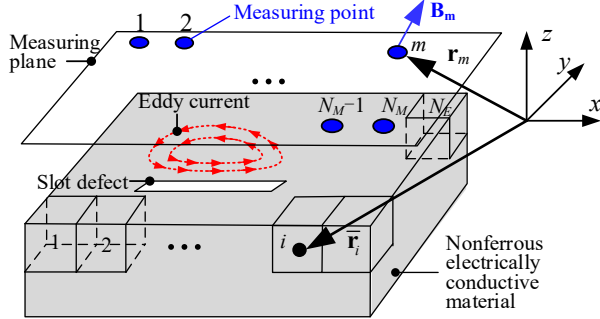


Figure 1. Schematics of physical field reconstruction

### A. Measurement Models of Physical Fields

The measurement models satisfy the following assumptions: 1) the electric conductors obey Ohm's law. 2) The system operates in magneto quasi-static (MQS) conditions;  $2\pi Lf \ll 1/\sqrt{\mu\epsilon} \approx 3 \times 10^8$  where  $\epsilon$  and  $\mu$  are the permittivity and magnetic permeability of the conductor;  $f$  is the operating frequency, and  $L$  is the characteristic length. As shown in Fig. 1, the conductor is evenly discretized into  $N_E$  elements with their locations denoted by the displacement vector  $\mathbf{r}_i$  where  $i=1 \dots N_E$ , and the MFD are measured at  $N_M$  neighboring points.

#### A.1 Conductivity

The measurement vector  $\mathbf{B}_E$  is a nonlinear function of the  $N_E$  elemental conductivities,  $\mathbf{B}_E(\sigma_1, \dots, \sigma_{N_E})$ . Linearized about its mean value  $(\bar{\sigma}, \bar{\mathbf{B}}_E)$ , the measurement model of the conductivity distribution is formulated in (1) where  $\boldsymbol{\eta}_1$  accounts for the errors in the measured deviation vector  $\hat{\mathbf{B}}_E (= \mathbf{B}_E - \bar{\mathbf{B}}_E)$ :

$$\hat{\mathbf{B}}_E = [\mathbf{W}] \hat{\boldsymbol{\sigma}} + \boldsymbol{\eta}_1 \quad (1)$$

where  $\hat{\boldsymbol{\sigma}} (= \boldsymbol{\sigma} - \bar{\boldsymbol{\sigma}}) = [\hat{\sigma}_1 \dots \hat{\sigma}_i \dots \hat{\sigma}_{N_E}]^T$ ;

$$\text{and } [\mathbf{W}] = \begin{bmatrix} \frac{\partial \mathbf{B}_E}{\partial \sigma_1} & \frac{\partial \mathbf{B}_E}{\partial \sigma_2} & \dots & \frac{\partial \mathbf{B}_E}{\partial \sigma_{N_E}} \end{bmatrix}.$$

In (1),  $\hat{\boldsymbol{\sigma}}$  is the conductivity deviation vector; and  $[\mathbf{W}]$  is the Jacobian matrix with its column vector given by

$$\frac{\partial \mathbf{B}_E}{\partial \sigma_i} = \frac{1}{\Delta \sigma} (\tilde{\mathbf{B}}_{Ei} - \bar{\mathbf{B}}_E) \quad (2)$$

where  $\tilde{\mathbf{B}}_{Ei}$  represents the measured MFD assuming uniform conductivity  $\bar{\sigma}$  except at the  $i^{\text{th}}$  element where  $\sigma_i = \bar{\sigma} + \Delta \sigma$ . Given the external source, the conductor shape and operating point  $(\bar{\sigma}, \bar{\mathbf{B}}_E)$ ,  $[\mathbf{W}]$  can be pre-calculated, for example, by means of a finite element analysis (FEA).

#### A.2 Hidden Geometrical Feature

The hidden geometrical features (such as cavity defects within which no eddy current is generated) can be treated as "one of more missing elements". The ECD-generated MFD can be regarded as the net effect of all the conductor elements. Thereby, the reconstruction of the hidden geometrical feature is formulated in (3) where  $\boldsymbol{\eta}_2$  accounts for the errors in the data  $\hat{\mathbf{B}}_E$ , and  $\bar{\mathbf{B}}_E$  corresponds to the data for the same metallic plate but homogenous with constant conductivity:

$$\hat{\mathbf{B}}_E = [\mathbf{Q}] \mathbf{V} + \boldsymbol{\eta}_2 \quad (3)$$

where  $[\mathbf{Q}] = [\mathbf{q}_1 \ \mathbf{q}_2 \ \dots \ \mathbf{q}_i \ \dots \ \mathbf{q}_{N_E}]$  is the sensitivity matrix with its column vector  $\mathbf{q}_i$  in  $[\mathbf{Q}]$  ( $\in \mathbb{R}^{3N_M \times N_E}$ ) given by (4) where  $\tilde{\mathbf{B}}_{Ei}$  is the MFD measurements assuming a homogeneous plate (with constant conductivity) except the  $i^{\text{th}}$  element:

$$\mathbf{q}_i (\in \mathbb{R}^{3N_M \times 1}) = \tilde{\mathbf{B}}_{Ei} - \bar{\mathbf{B}}_E \quad (4)$$

Given the external source and conductor shape, the sensitivity matrix  $[\mathbf{Q}]$  can be pre-calculated using FEA. In (4), the existent probability  $\mathbf{V}$  ( $\in \mathbb{R}^{N_E \times 1}$ ) characterizes the interior geometry of the conductor. Each element in  $\mathbf{V}$  has a value between 0 (conductive) and 1 (non-conductive).

#### B. Solutions to Inverse Problem

The reconstruction of a physical field from limited discrete MFD measurements is essentially an inverse problem that solves for the field of interest  $\mathbf{X}$  ( $\in \mathbb{R}^{m \times 1}$ ) using (1) and (3) given measurement vector  $\mathbf{Y}$  ( $\in \mathbb{R}^{n \times 1}$ ) and  $[\mathbf{K}]$  with noise  $\boldsymbol{\eta}$ :

$$\mathbf{Y} = [\mathbf{K}] \mathbf{X} + \boldsymbol{\eta} \quad (5)$$

For a noise-free system with nonsingular  $[\mathbf{K}]$  and  $n \geq m$ ,  $\mathbf{X}$  can be solved from the pseudoinverse solutions (6):

$$\mathbf{X} \approx ([\mathbf{K}]^T [\mathbf{K}])^{-1} [\mathbf{K}]^T \mathbf{Y} \quad (6)$$

However,  $n < m$  in practice when reconstructing a physical field from limited measurement data. As a result,  $[\mathbf{K}]$  becomes increasingly ill-conditioned as measurement cost increases. The pseudo-inverse solutions (6) fail as the noise  $\boldsymbol{\eta}$  in  $\mathbf{Y}$  may be greatly amplified. To address these problems in practice, Tikhonov regularization [19] commonly used in machine learning is employed to solve the ill-posed problem:

$$\mathbf{X}_\alpha = [\mathbf{R}_\alpha] \mathbf{Y} \quad (7a)$$

$$\text{where } [\mathbf{R}_\alpha] = ([\mathbf{K}]^T [\mathbf{K}] + \alpha \mathbf{I})^{-1} [\mathbf{K}]^T \quad (7b)$$

In (7b),  $\alpha$  (with range  $[0, 1]$ ) is the regularization parameter to control the noise effects and prevent overfitting. A small  $\alpha$  will result in the inadequate filtering of the noise and highly oscillatory solution. On the other hand, most components of the solution are also filtered out with overly smooth solutions. In this work,  $\alpha$  is determined using the L-Curve graphical method [20][21] to tradeoff between noise filtering and solution smoothness which requires no prior information about noise.

### III. NUMERICAL RESULT AND DISCUSSION

The effectiveness of the machine perception method for reconstructing the electrical conductivity field and the hidden geometrical features has been numerically investigated. As schematically illustrated in Fig. 2(a), an EM is placed above (with its axis perpendicular to) the aluminum plate. The parameters are detailed in TABLE I where  $I_0$  represents the amplitude of the excitation current flowing through the EM coil of  $N$  turns;  $z_C$  and  $z_S$  are the distances from the top surface of the plate and the MFD sensor respectively; and  $f$  is the coil excitation frequency. The conductor is evenly divided into small elements with the size of  $2 \times 2 \times 1$  (mm).

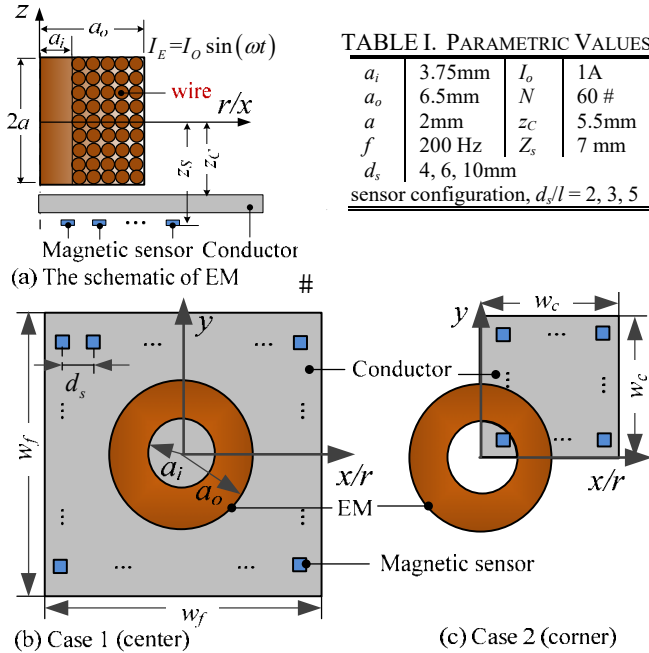


Figure 2. Two cases for simulations

As an ill-posed inverse problem, the sensor configuration characterized by the ratio  $d_s/l$  (where  $l$  is the smallest dimension of the elements) has a critical effect on the conductivity estimation performance and on the resolution and accuracy of feature detection. Three sensor configurations ( $d_s/l = 2, 3, 5$ ) are used in the numerical investigation.

#### A. Conductivity Estimation

For the estimation of the continuous electrical conductivity distribution, the EM is placed at the corner of the conductive plate (size: 30mm $\times$ 30mm $\times$ 1mm) as shown in Fig. 2 (c).

The sensors are evenly spaced with interval  $d_s = 4$ mm (or  $d_s/l = 2$ ). With low excitation frequency and thin conductor thickness, the phenomenon of the skin effect is not obvious. The estimated results are compared with the pre-set electrical conductivity.

The “MFD measurements” were simulated using FEA, and the conductivity distribution of the plate is assumed to have the form:

$$\sigma(x, y) = \sigma_0 \left[ 1 + \frac{q_0}{0.03^2} (x^2 + y^2) \right] \quad (8)$$

where  $\sigma_0 (= 3.774 \times 10^7 \text{S/m})$  is the conductivity of aluminum;  $q_0$  is a constant that determines the variation of the preset conductivity distribution. The conductivity is estimated for two types of variations:

Large Variation (LV):  $q_0 = 0.3$

Small Variation (SV):  $q_0 = 0.03$

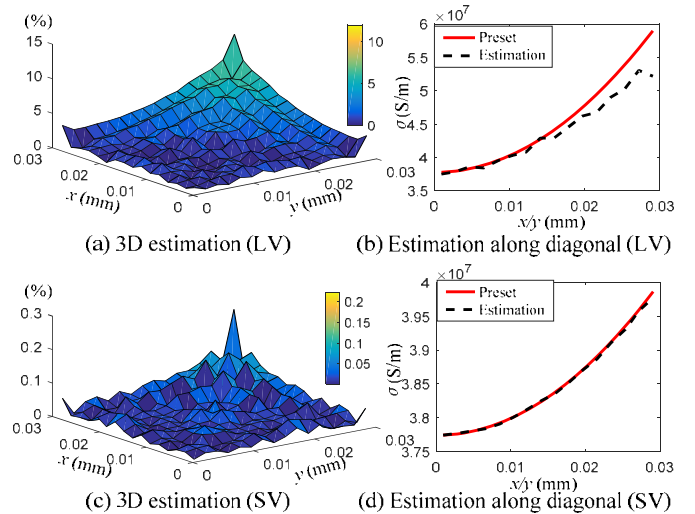


Figure 3. Conductivity estimation with boundary effects

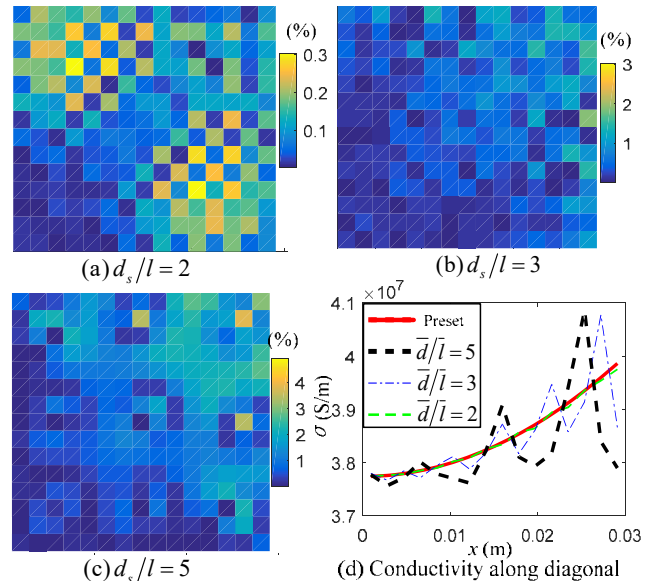


Figure 4. Conductivity estimation with different ratio

The simulated results are compared with the preset conductivity distribution in Fig. 3. Fig. 4 manifests simulation results with different the sensor configurations.

From Fig. 3, the conductivity of small variation is much better than that of large variation. Even with boundary effects, the conductivity still can be successfully estimated, especially for the small variation. As illustrated in Fig. 4, the estimation error dramatically increases as the sensor interval increases.

### B. Hidden Geometrical Feature Detection

Two common geometrical (cylindrical and slot) features are used to illustrate the application of machine perception method to characterize the hidden geometrical features inside the conductive plate (size: 60mm×60mm×4mm):

**Cylindrical feature** (Figs. 5a, b) where three different types of a 6mm-radius circular hole are located at (10, 10) mm on the x-y plane; through hole, hidden cavity and blind hole.

**Slot feature** (Figs. 5c, d) where three different types of a 20mm×2mm slot are located at (0, -10) mm on the x-y plane; through slot, hidden cavity and blind slot.

Results are summarized in Figs. 6 and 7.

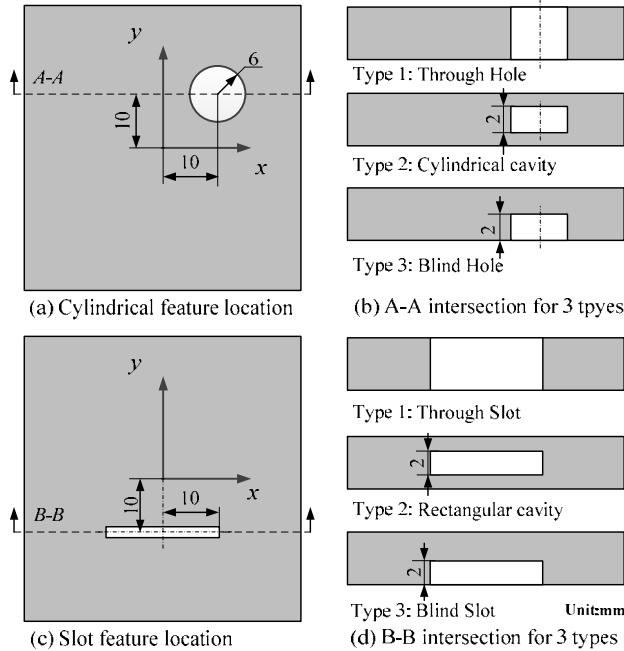


Figure 5. Cylindrical and slot defect setting

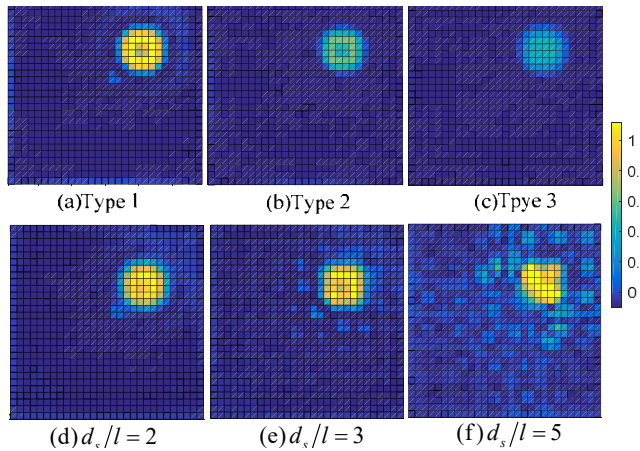


Figure 6. Cylindrical feature estimation

The geometrical features are successfully identified as shown in Figs. 6 and 7, where the elements with yellow color indicate through features; and the elements with light blue indicate internal features. As illustrated in Figs. 6 and 7, the quality of the estimation decreases as the ratio  $d_s/l$  increases. The edge of the estimated defect becomes blurry and some shadows come out close to the estimated defects.

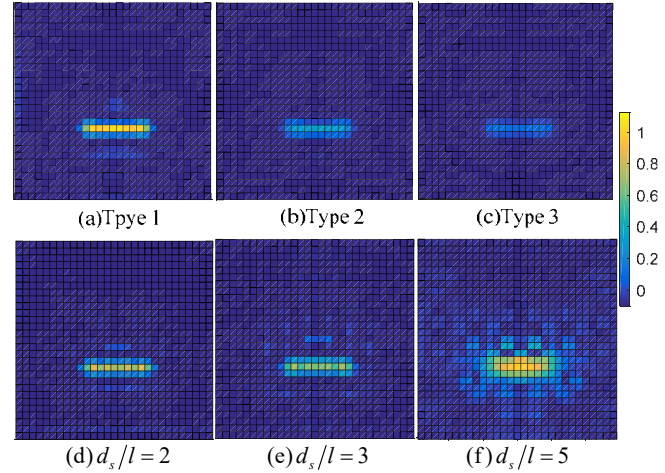
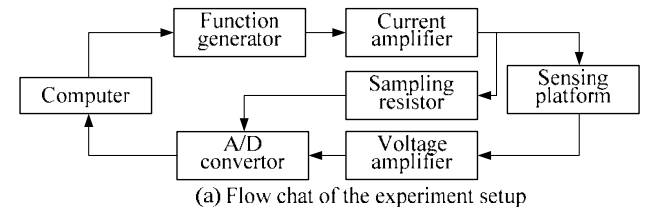


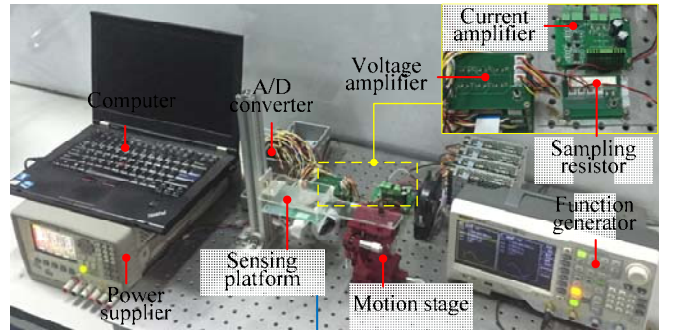
Figure 7. Slot defect estimation

## IV. EXPERIMENTAL RESULTS AND DISCUSSION

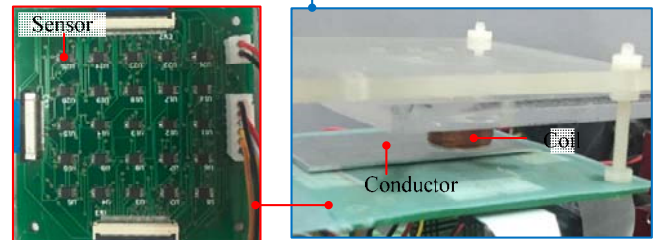
Several experiments are performed on the experimental setup as shown in Fig. 8 to verify the methods of physical field reconstruction using MFD measurements.



(a) Flow chat of the experiment setup



(b) Experimental setup overview



(c) Sensor board

(d) Sensing platform

Figure 8. Experimental setup for eddy current reconstruction

### A. Experimental Setup

As illustrated in Fig. 8, the experimental setup consists of an EM (parametric values given in TABLE II), an array of analog magnetic sensors mounted on a printed circuit board (PCB), voltage/current amplifiers, analog-to-digital (AD) converters, a function generator, a 3D precision motion stage and a computer. An NI (National Instruments) device cDAQ-9178 with three modules NI-9205 is utilized to simultaneously acquire voltage data from all the analog channels, which communicate with a host computer through a USB port. The EM is attached to the motion stage so that it can be accurately moved to any specified location. The voltage amplifiers are used to amplify the sensor output before data collection, whereas the current amplifier supplies a sinusoidal current flowing through the EM with a sinusoidal input through a DA converter. By measuring the voltage of the sampling resistor, the excitation current can be accurately determined.

Considering the manufacturing feasibility of the sensor board and the effects of space spacing  $d_s$  on the reconstruction error, the sensor array consists of  $5 \times 5$  analog magnetic sensors (HMC1052 with two sensitive axes) with  $d_s = 10\text{mm}$ . For the experiments, the sinusoidal current frequency is set to 1kHz whereas the sampling rate for each channel is set to 10kHz. As shown in Fig. 8(d), to avoid sensor saturation caused by the primary magnetic field generated by the EM, the conductor is placed between the sensor board and EM.

TABLE II. PARAMETERS OF THE COIL

|       |         |       |       |
|-------|---------|-------|-------|
| $a_i$ | 6.5mm   | $I_o$ | 1A    |
| $a_o$ | 7.7mm   | $N$   | 60 #  |
| $a$   | 3.65 mm | $f$   | 1 kHz |

### B. Results and Conductivity Estimation

Two types of 1 mm-thick plates (with different dimensions of  $56 \times 56\text{mm}$  and  $56 \times 28\text{mm}$ ) were utilized for the conductivity estimation tests. For each type, two different plates made of aluminum alloy 6061 and 7075 with the conductivity  $2.16 \times 10^7 \text{ S/m}$  and  $1.903 \times 10^7 \text{ S/m}$  respectively were used in the investigation. To eliminate the effects of the error between the simulation and experiments,  $\bar{\mathbf{B}}_E$  is directly measured in the experiments using the conductor with known conductivity instead of using simulation results.

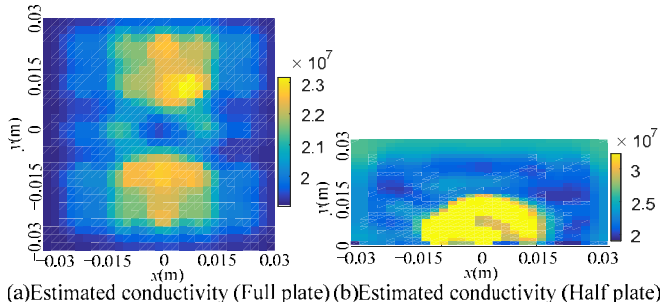


Figure 9. Conductivity estimation for 6061

Since aluminum alloy 6061 and 7075 have relatively close conductivity, using the information of either material, the conductivity of the other can be estimated. Assume the conductivity of 7075 is known and that of 6061 need to be estimated. With the evenly divided element size  $1.86 \times 1.86 \times 1(\text{mm})$ , the conductivity of full and half plates

made of aluminum alloy 6061 is estimated in Fig. 9. The following are some observations drawn from the experimental results in Fig. 9.

- The conductivity of the full and half plates made of 6061 can be estimated by measuring MFD generated by the eddy current.
- The estimation errors in the area closed to the boundary and the center of EM are relatively large due to the low SNR.
- As shown in Fig. 9(b), the conductivity can be accurately estimated in the area closed to the boundary on the  $x$  axis regardless of boundary effects.
- The measurement errors of MFD and the dimension error of the experimental setup, sensor configuration and EM may also lead to the error of conductivity estimation.

### C. Results of Hidden Geometrical Feature Detection

Four types of geometrical features within a ( $56\text{mm} \times 56\text{mm} \times 3\text{mm}$ ) plate as shown in Fig. 10 were utilized in the experiment. The size and location of geometrical features, which are the 1.5mm-deep blind hole and slot and the through hole and slot, are manifested in Fig. 10. The blind hole and slot have the same corresponding size and location as the through hole and slot.

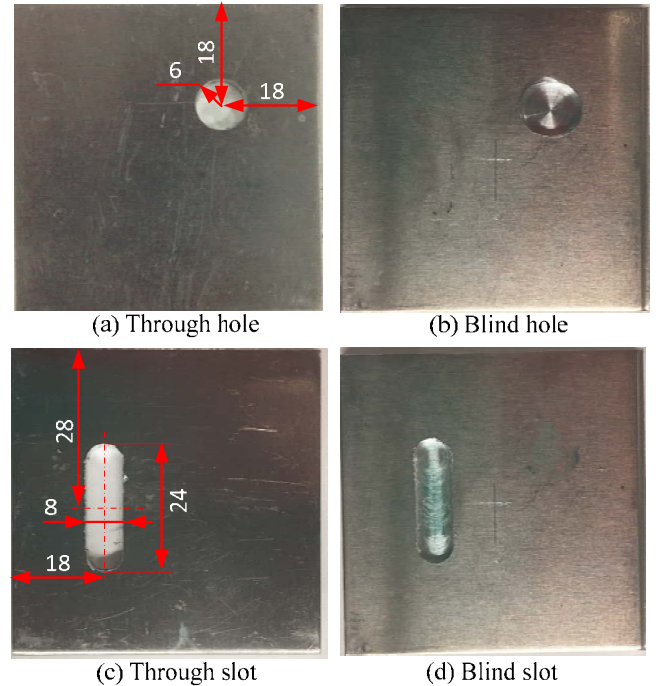


Figure 10. Cavity defects samples

With the evenly divided element size of  $1.4 \times 1.4 \times 1(\text{mm})$ , the experimental results are manifested in Fig. 11 from which the following observations can be drawn:

- The geometrical features (hole and slot) can be detected by using MFD measurements.
- The values of corresponding elements in the areas of through features are approximately equal to 1 whereas those of the elements in internal features (interpreted using blind hole and slot) are approximately equal to the values less than 1.

- Besides the actual features, there are several shadows which have similar shapes of the defects in the results and seem to follow certain pattern as shown in Fig. 11. The intensity of the shadows is much smaller than that of actual cavity defects.
- Inspired by the simulation results of the sensor configuration effects, the phenomenon of shadows may be the results of MFD measurement errors and the errors of Regularization method due to the large ratio  $d_s/l$ .

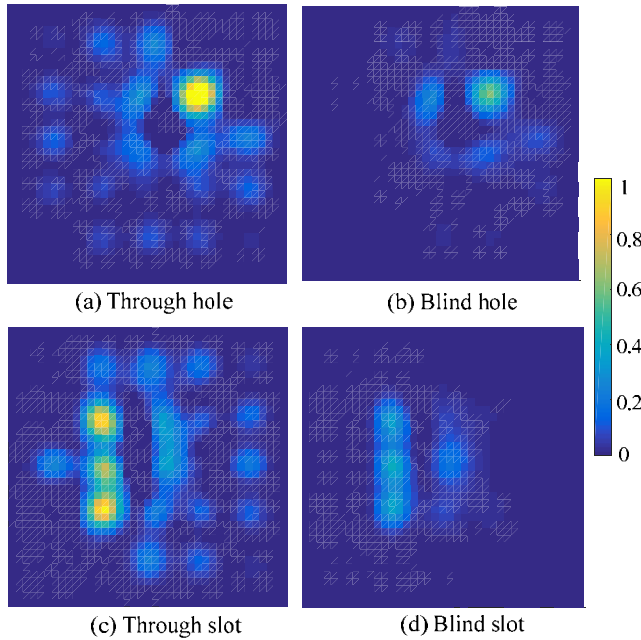


Figure 11. Cavity defects estimation

## V. CONCLUSION

A new method of machine perception based on physical field reconstruction using finite measurements of magnetic field for intelligent manufacturing has been presented. By discretizing the conductor, the measurement models of the physical fields based on the discrete MFD measurements have been linearly established. The models transform the physical field reconstruction to a linear inverse problem, which is solved using the Tikhonov regularization method. The effectiveness and accuracy of the reconstruction method are demonstrated numerically and experimentally. The conductivity field and hidden geometrical feature of aluminum plates are estimated along with a numerical analysis that investigates the effects of sensor configuration on the reconstruction. The physical fields are successfully reconstructed using a self-built experimental setup consisting of a magnetic sensor array, an air-cored EM and associated hardware. The method for conductivity field reconstruction has the potential to be extended for other continuous physical field reconstructions.

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