

Flux Field Formulation and Back-Iron Analysis of Tubular Linear Machines

Liang Yan¹, Jie Hu¹, Nan Yao¹, Zongxia Jiao², I-Ming Chen³, *Fellow, IEEE*, and Kok-Meng Lee⁴, *Fellow, IEEE*

¹School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

²Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing 100191, China

³School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore 637098

⁴George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405 USA

In electromagnetic machines, ferromagnetic material is one of the key components that affect the magnetic field and thus the output performance significantly. The objective of this paper is to study the influence of ferromagnetic material on magnetic field distribution of tubular electromagnetic linear machines. Generic magnetic field models are obtained for linear machines with various structure designs based on magnetic vector potential and Bessel functions. The magnetic field model is validated with numerical results from finite element calculations. The comparison between them shows that the flux model fits closely with the numerical computation. The obtained analytical model can be employed to study the flux density distribution of machine designs with single-sided, double-sided, and non back irons. It shows that the use of back iron, especially for the internal side, helps to greatly enhance the radial component of magnetic field. This component contributes to the axial force generation of tubular linear machines. The study could be useful for the design optimization of permanent-magnet linear machines.

Index Terms—Back iron, electromagnetic actuator, linear machine, magnetic field.

I. INTRODUCTION

AN electromagnetic linear machine is a special rotary machine that has its stator and rotor “unrolled” so that instead of producing a torque (rotation) it produces a linear force along its length. It can generate linear motions directly without rotation-to-translation conversion mechanism, which significantly simplifies system structure and improves the power density. Because a gear box is no longer necessary, it could possibly increase the motion precision of mechanisms. Among various designs, the tubular linear machine with permanent magnets (PMs) has a number of distinctive features such as high force density and excellent servo characteristics [1]. Due to the fascinating advantages, it has wide applications in robotics [2], manufacturing [3], transportation [4], medical operation [5], and micro-manipulation [6]. There are numerous studies of electromagnetic linear machines that mainly focus on PM-pole patterns or coil arrangements. For example, Liu *et al.* designed a voice coil motor with a single pair of magnets to minimize the system size [7]. Wang *et al.* presented a linear PM machine with alternatively magnetization pattern in axial direction to increase the force output [8]. Kimoulakis *et al.* used skewed mounted PMs in a linear machine [9] to increase power efficiency. To enhance the magnetic loading capacity, a quasi-Halbach array was proposed for linear actuators in [10]. Besides PM arrangement, there are some studies on coil patterns. For example, Mutai *et al.* presented an actuator with concentric multiple coils [11]. These coils can either be polarized in the same direction, or in opposite directions for adjacent windings. The latter can produce high force in short motion range. Kim *et al.* proposed an electromagnetic linear actuator for quick latching [12]. Two sets of windings are arranged face to face to pull the armature in two

directions. Each set of windings includes two coils with opposite excitation.

However, studies have seldom been done on the pattern of back iron, although it is widely employed in electromagnetic motors to reduce the magnetic energy loss, improve the flux field density, and thus increase the force/torque output. Therefore, the objective of this paper is to study the effect of back iron pattern on the magnetic field distribution of linear machines. Various methods have been proposed to study the magnetic field of electromagnetic machines. One typical method is the finite element method (FEM) that employs the numerical technique to calculate the magnetic field by elements [13], [14] or automatic differentiation [15]. It simplifies the analytical process, and can achieve the result conveniently. However, it remains time-consuming and does not provide an insightful way to evaluate the influence of design parameters on the output performance. Meessen *et al.* proposed a semi-analytical formulation of magnetic scalar potential to study the magnetic field of linear motors with segmented quasi-Halbach arrays [16]. Another method based on Maxwell equations was proposed by Prieto *et al.* [17] to obtain the one-dimensional (1-D) magnetic component model for planar structures successfully. As an improvement, [18] presented an exact 2-D analytical model for predicting the armature reaction magnetic field in idealized structures of linear machines. Xiong *et al.* [19] have proposed an analytical method of the magnetic field for a linear PM synchronous machine by using the concept of magnetic charge. Wang *et al.* [20] and Lee *et al.* [21] have proposed magnetic field modeling methods based on Laplace’s equation in cylindrical and spherical coordinates. Prieto *et al.* [22] presented a procedure to obtain the energy in magnetic structures by applying the double 2-D methodology. Equivalent magnetic flux circuit was proposed by researchers to formulate the magnetic field of quite a few linear machines [23], [24]. In this study, a generic magnetic field modeling method based on vector potential in cylindrical coordinates and Bessel functions is proposed. It can be generalized for PM machine designs with different types of boundary conditions. Analytical flux

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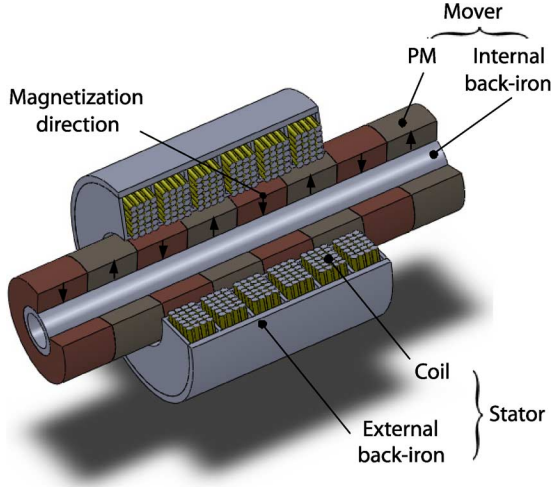


Fig. 1. Schematic drawing of electromagnetic linear machine.

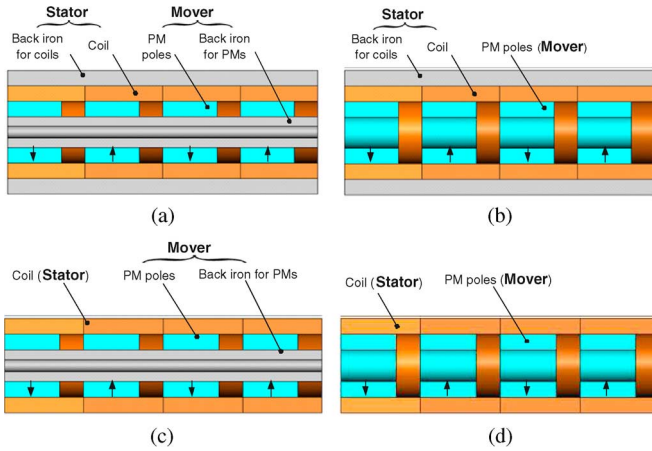


Fig. 2. Configurations of back iron in linear machines. (a) Double back irons; (b) external back iron; (c) internal back iron; (d) back iron free.

II. STRUCTURE AND OPERATING PRINCIPLE

The structure of the proposed linear machine is illustrated in Fig. 1. The mover is composed of PMs that produce magnetic flux field in the machine. The PMs are magnetized alternatively in radial direction. The stator consists of coils excited to generate axial force by interacting with the external magnetic field of PMs. Thus, the mover can translate in the axial direction. The stator and mover could be switched depending on what part we want to move. Sectional views of the linear machines are presented in Fig. 2. The major difference among the four figures in Fig. 2 is the arrangement of back irons. Specifically, Fig. 2(a) shows the linear motor with back irons in both stator and mover, Fig. 2(b) single-layer of back iron in stator, Fig. 2(c) single-layer of back iron in mover, and Fig. 2(d) without any back iron. In following sections, we will formulate the magnetic field of tubular linear machines analytically, and then gen-

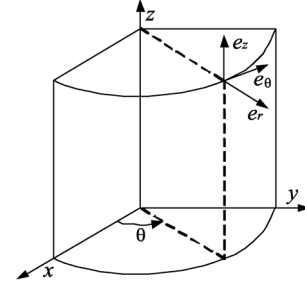


Fig. 3. Cylindrical coordinators for PM linear machines.

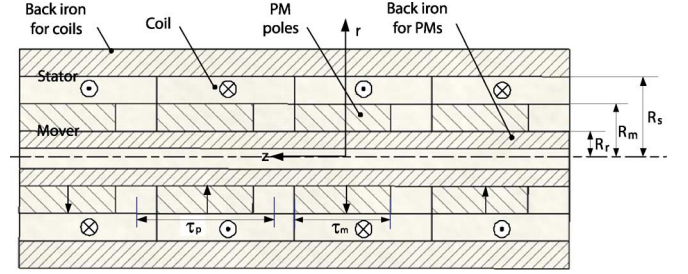


Fig. 4. Magnetization pattern and parameters in PM linear machines.

eralize the model for different boundary conditions for performance analysis.

III. GOVERNING EQUATIONS OF FLUX FIELD

A. Assumptions

- The following assumptions are made for subsequent study.
- The length of the linear machine is infinite, and thus the end effect of the magnetic field is ignorable.
 - The magnetic field is symmetrically distributed around the central axis.
 - The magnetic permeability of soft iron is infinite.
 - Eddy currents in stator and mover are ignorable.

B. Magnetic Property: Relations Between H and B

The motor space under study is divided into two regions based on their magnetic characteristics. The air space or coil space outside the mover is denoted as Region 1. The PM volume filled with rare-earth magnetic material is denoted as Region 2. The back iron with high permeability can enhance the flux density by reducing the magnetic energy loss. The magnetic field property of Region 1 and 2 is characterized by the relationship between magnetic field intensity, $\mathbf{H}$ (in A/m), and flux density, $\mathbf{B}$ (in Tesla), as

$$\mathbf{B}_1 = \mu_0 \mathbf{H}_1, \quad \mathbf{B}_2 = \mu_0 \mu_r \mathbf{H}_2 + \mu_0 \mathbf{M} \quad (1)$$

where μ_0 is permeability of free space with a value of $4\pi \times 10^{-7}$ H/m, dimensionless quantity μ_r is the relative permeability of permanent magnets, $\mathbf{M} = \mathbf{B}_{rem}/\mu_0$ is the residual magnetization vector in A/m, and $\mathbf{B}_{rem}$ is remanence.

C. Governing Equations

The governing equations of magnetic field, i.e. Laplace's equation and Poisson equation, are significant for the solution

of flux field. They are derived as follows. Magnetic field is a solenoid field or source-free field, i.e.,

$$\nabla \cdot \mathbf{B}_i = 0 \quad (2)$$

where $i = 1, 2$. There is a vector potential, $\mathbf{A}_i$, so that

$$\mathbf{B}_i = \nabla \times \mathbf{A}_i. \quad (3)$$

Therefore, the three components of $\mathbf{B} = (B_r, B_\theta, B_z)$ are

$$B_r = -\frac{\partial A_\theta}{\partial z}, \quad B_\theta = 0, \quad B_z = \frac{1}{r} \frac{\partial}{\partial r}(r A_\theta). \quad (4)$$

Under Coulomb gauge, we could have

$$\nabla \times \mathbf{B}_i = \nabla(\nabla \cdot \mathbf{A}_i) - \nabla^2 \mathbf{A}_i = -\nabla^2 \mathbf{A}_i. \quad (5)$$

1) *Laplace's Equation for Region 1:* The combination of Maxwell's equations and (1) gives

$$\nabla^2 \mathbf{A}_1 = -\mu_0 \mathbf{J}. \quad (6)$$

where $\mathbf{J}$ (A/m²) is current density. In this study, $\mathbf{J} = 0$. Therefore, the Laplace's equation for Region 1 is

$$\nabla^2 \mathbf{A}_1 = 0. \quad (7)$$

2) *Poisson's Equation for Region 2:* Similarly,

$$\nabla^2 \mathbf{A}_2 = -\mu_0 \nabla \times \mathbf{M}. \quad (8)$$

D. Governing Equations in Cylindrical Coordinators

Denote $\mathbf{A}_i = (A_{i,r}, A_{i,\theta}, A_{i,z})$. The left side of Laplace's and Poisson equations, $\nabla^2 \mathbf{A}_i = (\delta_{i,r}, \delta_{i,\theta}, \delta_{i,z})$, can be written in cylindrical coordinate system. The three components of $\nabla^2 \mathbf{A}_i$ are represented respectively as

$$\begin{aligned} \delta_{i,r} &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_{i,r}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_{i,r}}{\partial \theta^2} + \frac{\partial^2 A_{i,r}}{\partial z^2} - \frac{A_{i,r}}{r^2} \\ &\quad - \frac{2\partial A_{i,\theta}}{r\partial \theta} \\ \delta_{i,\theta} &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_{i,\theta}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_{i,\theta}}{\partial \theta^2} + \frac{\partial^2 A_{i,\theta}}{\partial z^2} - \frac{A_{i,\theta}}{r^2} \\ &\quad - \frac{2\partial A_{i,r}}{r\partial \theta} \\ \delta_{i,z} &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_{i,z}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_{i,z}}{\partial \theta^2} + \frac{\partial^2 A_{i,z}}{\partial z^2}. \end{aligned} \quad (9)$$

Because the magnetic field of the linear motors is axially symmetric, $\mathbf{A}_i$ has only one component, $A_{i,\theta}$, that is independent of θ . Because $A_{i,r}$ and $A_{i,z}$ are equal to zero, we have

$$\delta_{i,\theta} = \frac{\partial^2 A_{i,\theta}}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r A_{i,\theta}) \right), \quad \delta_{i,r} = \delta_{i,z} = 0. \quad (10)$$

$$\begin{cases} \frac{\partial^2 A_{1,\theta}}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r A_{1,\theta}) \right) = 0, \\ \frac{\partial^2 A_{2,\theta}}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r A_{2,\theta}) \right) = -\mu_0 N_\theta \end{cases} \quad (11)$$

where N_θ is the component of $\nabla \times \mathbf{M}$ in θ direction.

IV. SOLUTIONS TO MAGNETIC FIELD

A. Harmonic Expansion of Magnetization Vector

The magnetization vector, $\mathbf{M} = (M_r, M_\theta, M_z)$, is

$$\mathbf{M} = M_r \mathbf{e}_r + M_\theta \mathbf{e}_\theta + M_z \mathbf{e}_z \quad (12)$$

where M_r , M_θ and M_z are the components in r , θ and z directions respectively. It is known that $M_\theta = M_z = 0$ and M_r is only dependant on θ . Therefore, the three components N_r , N_θ and N_z of $\nabla \times \mathbf{M}$ are represented as

$$\begin{cases} N_r = \left(\frac{\partial M_z}{\partial \theta} - \frac{\partial r M_\theta}{\partial z} \right) \frac{1}{r} = 0 \\ N_\theta = - \left(\frac{\partial M_r}{\partial r} - \frac{\partial M_r}{\partial z} \right) = \frac{\partial M_r}{\partial z} \\ N_z = \left(\frac{\partial r M_\theta}{\partial r} - \frac{\partial M_r}{\partial \theta} \right) \frac{1}{r} = - \frac{\partial M_r}{\partial \theta} \frac{1}{r} = 0. \end{cases} \quad (13)$$

M_r is a function with period of $2\tau_p$, specifically

$$M_r = \begin{cases} -M_0 & -\tau_p \leq z < \frac{\tau_m}{2} - \tau_p, \quad \tau_p - \frac{\tau_m}{2} \leq z < \tau_p \\ 0 & \frac{\tau_m}{2} - \tau_p \leq z < -\frac{\tau_m}{2}, \quad \frac{\tau_m}{2} \leq z < \tau_p - \frac{\tau_m}{2} \\ M_0 & -\frac{\tau_m}{2} \leq z < \frac{\tau_m}{2} \end{cases} \quad (14)$$

where M_0 is the magnitude of magnetization vector $\mathbf{M}$. It can be expanded with Fourier series as

$$M_r^s = a_0 + \sum_{n=1}^{\infty} a_n \cos \left(\frac{2n\pi x}{T} \right) \quad (15)$$

where

$$a_n = 0, \quad a_n = \frac{4}{\tau_p} M_0 \frac{\tau_p}{n\pi} \sin \left(\frac{\tau_m}{2\tau_p} n\pi \right) \quad (16)$$

for n is even and odd number, respectively. Therefore, the Fourier expansion of M_r is

$$M_r^s = \sum_{n=1,2,\dots}^{\infty} 4 \left(\frac{B_{rem}}{\mu_0} \right) \frac{\sin \left[(2n-1) \frac{\pi}{2} \alpha_p \right]}{(2n-1)\pi} \cos(m_n z) \quad (17)$$

where $\alpha_p = \tau_m/\tau_p$, n is positive integer, and

$$m_n = (2n-1)\pi/\tau_p. \quad (18)$$

From (17) and (13), N_θ is obtained

$$N_\theta = - \sum_{n=1}^{\infty} 4 \left(\frac{B_{rem}}{\mu_0} \right) \frac{\sin \left[(2n-1) \frac{\pi}{2} \alpha_p \right]}{(2n-1)\pi} m_n \sin(m_n z). \quad (19)$$

Therefore,

$$-\mu_0 \nabla \times \mathbf{M} = \sum_{n=1}^{\infty} 4 B_{rem} \frac{\sin \left[(2n-1) \frac{\pi}{2} \alpha_p \right]}{\tau_p} \sin(m_n z). \quad (20)$$

Let $P_n = (4/\tau_p) B_{rem} \sin[(2n-1)(\pi/2)\alpha_p]$. Equation (20) is reduced

$$-\mu_0 \nabla \times \mathbf{M} = \sum_{n=1,2,\dots}^{\infty} P_n \sin(m_n z). \quad (21)$$

Substituting (21) into (11) results in

$$\begin{cases} \frac{\partial^2 A_{1,\theta}}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r A_{1,\theta}) \right) = 0, \\ \frac{\partial^2 A_{2,\theta}}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r A_{2,\theta}) \right) = \sum_{n=1,2,\dots}^{\infty} P_n \sin(m_n z). \end{cases} \quad (22)$$

B. Solutions to Laplace's and Poisson Equations

1) *Boundary Conditions:* The boundary conditions for the linear PM machines are

$$\begin{aligned} B_{1z}|_{r=R_s} &= 0; \quad B_{2z}|_{r=R_r} = 0; \\ B_{1r}|_{r=R_m} &= B_{2r}|_{r=R_m}; \quad H_{1z}|_{r=R_m} = H_{2z}|_{r=R_m}. \end{aligned} \quad (23)$$

The boundary conditions can be employed to solve the magnetic vector potential in governing equations, and thus obtain the flux field distribution.

C. Solutions to Flux Density Distribution

The solutions to Laplace's and Poisson equations are

$$A_{1\theta} = \sum_{n=1}^{\infty} [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \sin(m_n z), \quad (24)$$

$$\begin{aligned} A_{2\theta} = \sum_{n=1}^{\infty} \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \sin(m_n z) \right. \\ \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \sin(m_n z) \right\}. \end{aligned} \quad (25)$$

From (4), (24), and (25), we can have the solutions of magnetic flux density from vector potential as follows:

$$\begin{aligned} B_{r1} = \sum_{n=1}^{\infty} -m_n [a_{1n}I_1(m_n r) \\ + b_{1n}K_1(m_n r)] \cos(m_n z), \end{aligned} \quad (26)$$

$$\begin{aligned} B_{r2} = \sum_{n=1}^{\infty} -m_n \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \cos(m_n z) \right. \\ \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \cos(m_n z) \right\} \end{aligned} \quad (27)$$

$$B_{z1} = \sum_{n=1}^{\infty} m_n [a_{1n}I_0(m_n r) - b_{1n}K_0(m_n r)] \sin(m_n z) \quad (28)$$

$$\begin{aligned} B_{z2} = \sum_{n=1}^{\infty} m_n \left\{ [a_{2n}I_0(m_n r) - b_{2n}K_0(m_n r)] \sin(m_n z) \right. \\ \left. + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n r)}{m_n^2} P_n \sin(m_n z) \right\}. \end{aligned} \quad (29)$$

D. Determination of Coefficiencies

1) *Magnetic Field for Double-Sided Back Irons:* From the boundary conditions ((23)), the following equations can be obtained to calculate the coefficients in magnetic flux model in (26)–(29):

$$\begin{aligned} b_{1n} &= \frac{I_0(m_n R_s)}{K_0(m_n R_s)} a_{1n}, \\ b_{2n} &= \frac{1}{K_0(m_n R_r)} \\ &\times \left[\frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_r)}{m_n^2} P_n + a_{2n}I_0(m_n R_r) \right], \end{aligned}$$

$$\begin{aligned} a_{1n}I_1(m_n R_m) + b_{1n}K_1(m_n R_m) \\ = a_{2n}I_1(m_n R_m) + b_{2n}K_1(m_n R_m) \\ + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n R_m)}{m_n^2} P_n, \\ \mu_r [a_{1n}I_0(m_n R_m) - b_{1n}K_0(m_n R_m)] \\ = [a_{2n}I_0(m_n R_m) - b_{2n}K_0(m_n R_m)] \\ + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_m)}{m_n^2} P_n. \end{aligned}$$

2) *Magnetic Field for External Back Iron:* The boundary conditions for magnetic field of linear machine design with external back iron is

$$\begin{aligned} B_{z1}|_{r=R_s} &= 0, \quad B_{z3}|_{r=0} \neq \infty, \\ B_{1r}|_{r=R_m} &= B_{2r}|_{r=R_m}, \quad H_{1z}|_{r=R_m} = H_{2z}|_{r=R_m}, \\ B_{2r}|_{r=R_r} &= B_{3r}|_{r=R_r}, \quad H_{2z}|_{r=R_r} = H_{3z}|_{r=R_r}. \end{aligned} \quad (30)$$

From external back-iron pattern, the machine space is divided into three regions, Region 1 for air space in the core, Region 2 for PM, and Region 3 for winding area. Following the same flux field deriving process for double-sided back irons, the magnetic vector potential is obtained as

$$\begin{aligned} A_{1\theta} &= \sum_{n=1}^{\infty} [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \sin(m_n z), \\ A_{2\theta} &= \sum_{n=1}^{\infty} \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \sin(m_n z) \right. \\ &\quad \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \sin(m_n z) \right\}, \\ A_{3\theta} &= \sum_{n=1}^{\infty} [a_{3n}I_1(m_n r) + b_{3n}K_1(m_n r)] \sin(m_n z). \end{aligned}$$

Therefore, the magnetic field is computed from

$$B_{r1} = \sum_{n=1}^{\infty} -m_n [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \cos(m_n z) \quad (31)$$

$$\begin{aligned} B_{r2} = \sum_{n=1}^{\infty} -m_n \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \cos(m_n z) \right. \\ \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \cos(m_n z) \right\} \end{aligned} \quad (32)$$

$$B_{r3} = \sum_{n=1}^{\infty} -m_n [a_{3n}I_1(m_n r) + b_{3n}K_1(m_n r)] \cos(m_n z) \quad (33)$$

$$B_{z1} = \sum_{n=1}^{\infty} m_n [a_{1n}I_0(m_n r) - b_{1n}K_0(m_n r)] \sin(m_n z) \quad (34)$$

$$\begin{aligned} B_{z2} = \sum_{n=1}^{\infty} m_n \left\{ [a_{2n}I_0(m_n r) - b_{2n}K_0(m_n r)] \sin(m_n z) \right. \\ \left. + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n r)}{m_n^2} P_n \sin(m_n z) \right\} \end{aligned} \quad (35)$$

$$B_{z3} = \sum_{n=1}^{\infty} m_n [a_{3n}I_0(m_n r) - b_{3n}K_0(m_n r)] \sin(m_n z). \quad (36)$$

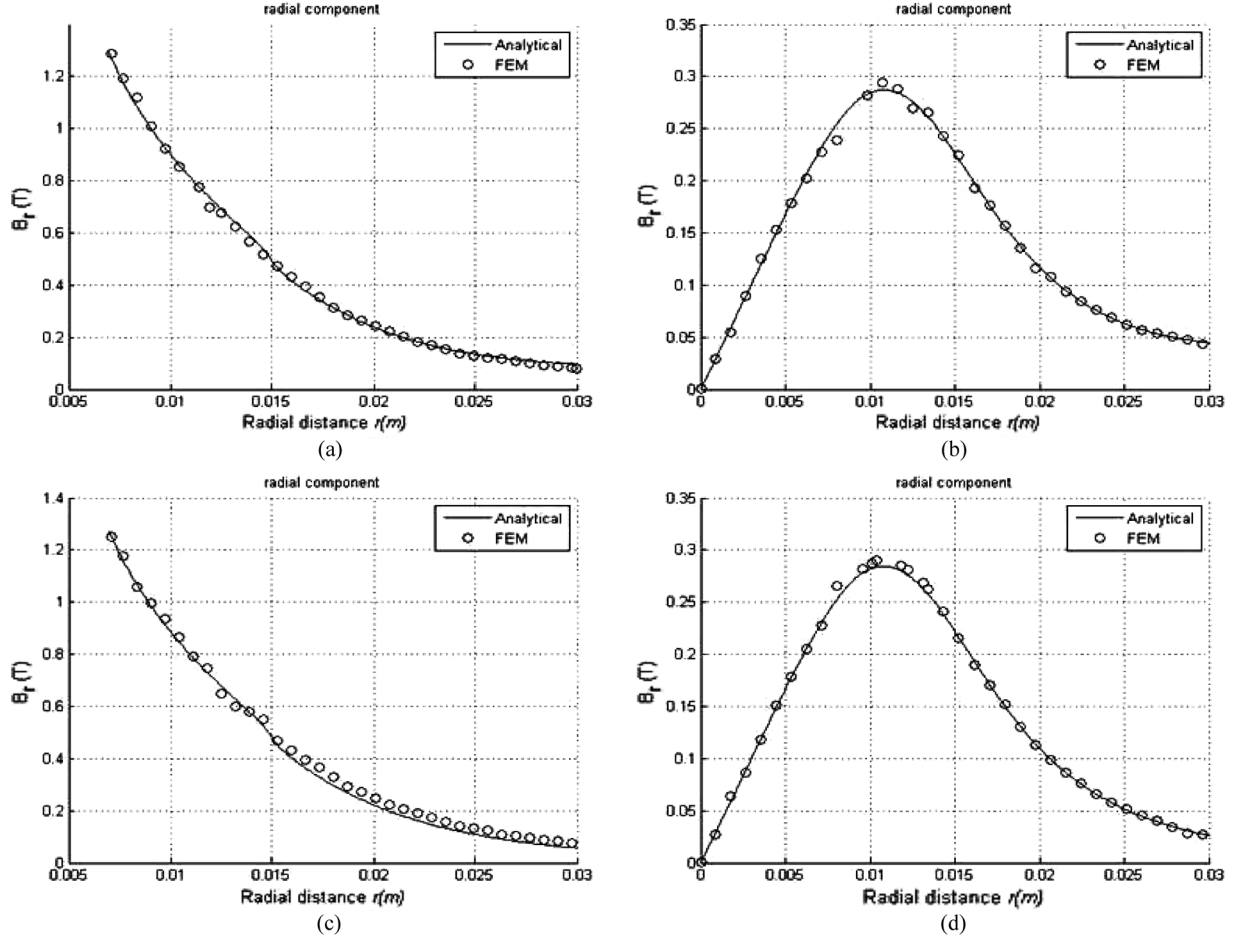


Fig. 5. B_r vs. radial distance r . (a) Double-sided back-irons; (b) external back-iron; (c) internal back-iron; (d) back-iron free.

The coefficients in the flux model can be calculated with

$$\begin{aligned}
 a_{1n}I_0(m_n R_s) - b_{1n}K_0(m_n R_s) &= 0, \quad b_{3n} = 0, \\
 a_{1n}I_1(m_n R_m) + b_{1n}K_1(m_n R_m) \\
 &= a_{2n}I_1(m_n R_m) + b_{2n}K_1(m_n R_m) \\
 &+ \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n R_m)}{m_n^2} P_n, \\
 \mu_r [a_{1n}I_0(m_n R_m) - b_{1n}K_0(m_n R_m)] \\
 &= [a_{2n}I_0(m_n R_m) - b_{2n}K_0(m_n R_m)] \\
 &+ \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_m)}{m_n^2} P_n, \\
 [a_{2n}I_1(m_n R_r) + b_{2n}K_1(m_n R_r)] \\
 &+ \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n R_r)}{m_n^2} P_n \\
 &= a_{3n}I_1(m_n R_r) + b_{3n}K_1(m_n R_r), \\
 \mu_r [a_{3n}I_0(m_n R_r) - b_{3n}K_0(m_n R_r)] \\
 &= [a_{2n}I_0(m_n R_r) - b_{2n}K_0(m_n R_r)] \\
 &+ \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_r)}{m_n^2} P_n.
 \end{aligned}$$

3) *Magnetic Field for Internal Back Iron*: The boundary conditions for machine designs with internal back iron are

$$\begin{aligned}
 B_{r1}|_{r=\infty} &= 0, \quad B_{z2}|_{r=R_r} = 0, \\
 B_{1r}|_{r=R_m} &= B_{2r}|_{r=R_m}, \quad H_{1z}|_{r=R_m} = H_{2z}|_{r=R_m}. \quad (37)
 \end{aligned}$$

Thus, the magnetic vector potential for the PM and coil areas is formulated as

$$\begin{aligned}
 A_{1\theta} &= \sum_{n=1}^{\infty} [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \sin(m_n z), \\
 A_{2\theta} &= \sum_{n=1}^{\infty} \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \sin(m_n z) \right. \\
 &\quad \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \sin(m_n z) \right\}.
 \end{aligned}$$

As a result, the magnetic field is formulated as

$$B_{r1} = \sum_{n=1}^{\infty} -m_n [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \cos(m_n z) \quad (38)$$

$$\begin{aligned}
 r_2 &= \sum_{n=1}^{\infty} -m_n \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \cos(m_n z) \right. \\
 &\quad \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \cos(m_n z) \right\} \quad (39)
 \end{aligned}$$

$$B_{z1} = \sum_{n=1}^{\infty} m_n [a_{1n}I_0(m_n r) - b_{1n}K_0(m_n r)] \sin(m_n z) \quad (40)$$

$$\begin{aligned}
 B_{z2} &= \sum_{n=1}^{\infty} m_n \left\{ [a_{2n}I_0(m_n r) - b_{2n}K_0(m_n r)] \sin(m_n z) \right. \\
 &\quad \left. + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n r)}{m_n^2} P_n \sin(m_n z) \right\}. \quad (41)
 \end{aligned}$$

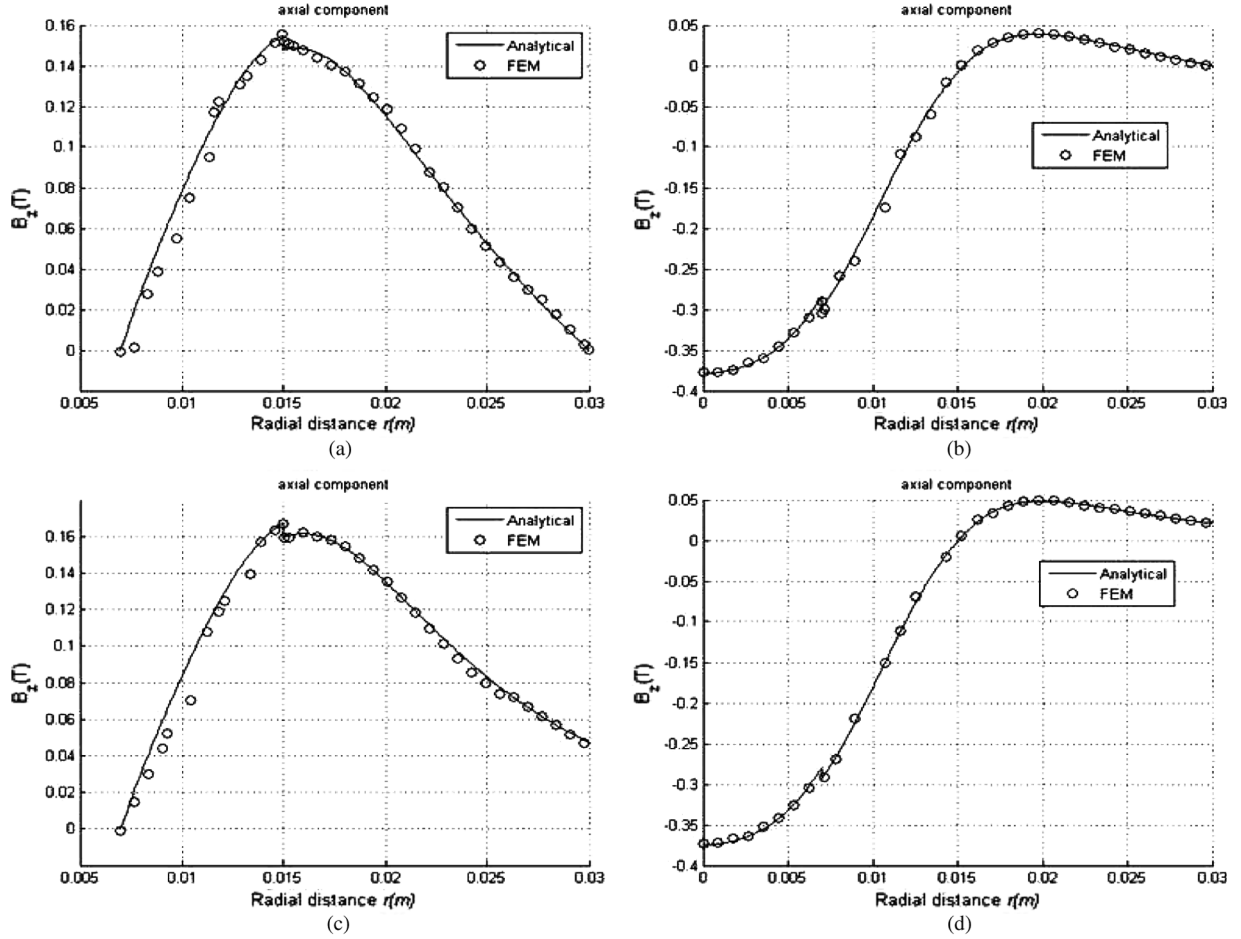


Fig. 6. B_z vs. radial distance r . (a) Double-sided back-irons; (b) external back-iron; (c) internal back-iron; (d) back-iron free.

The coefficients in the flux model are determined from

$$\begin{aligned}
 & [a_{2n}I_0(m_n R_r) - b_{2n}K_0(m_n R_r)] + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_r)}{m_n^2} P_n \\
 & = 0, \\
 & a_{1n}I_1(m_n R_m) + b_{1n}K_1(m_n R_m) \\
 & = a_{2n}I_1(m_n R_m) + b_{2n}K_1(m_n R_m) \\
 & + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n R_m)}{m_n^2} P_n, \quad a_{1n} = 0 \\
 & \mu_r [a_{1n}I_0(m_n R_m) - b_{1n}K_0(m_n R_m)] \\
 & = [a_{2n}I_0(m_n R_m) - b_{2n}K_0(m_n R_m)] \\
 & + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_m)}{m_n^2} P_n.
 \end{aligned}$$

4) *Magnetic Field for Back-Iron Free:* The boundary conditions for machine design without back irons are

$$\begin{aligned}
 & B_{r1}|_{r=\infty} = 0, \quad B_{z3}|_{r=0} \neq \infty, \\
 & B_{1r}|_{r=R_m} = B_{2r}|_{r=R_m}, \quad H_{1z}|_{r=R_m} = H_{2z}|_{r=R_m}, \\
 & B_{2r}|_{r=R_r} = B_{3r}|_{r=R_r}, \quad H_{2z}|_{r=R_r} = H_{3z}|_{r=R_r}. \quad (42)
 \end{aligned}$$

Therefore, the magnetic vector potential is

$$A_{1\theta} = \sum_{n=1}^{\infty} [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \sin(m_n z),$$

$$\begin{aligned}
 A_{2\theta} &= \sum_{n=1}^{\infty} \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \sin(m_n z) \right. \\
 &\quad \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \sin(m_n z) \right\}, \\
 A_{3\theta} &= \sum_{n=1}^{\infty} [a_{3n}I_1(m_n r) + b_{3n}K_1(m_n r)] \sin(m_n z).
 \end{aligned}$$

The magnetic field is obtained as follows:

$$B_{r1} = \sum_{n=1}^{\infty} -m_n [a_{1n}I_1(m_n r) + b_{1n}K_1(m_n r)] \cos(m_n z) \quad (43)$$

$$\begin{aligned}
 B_{r2} &= \sum_{n=1}^{\infty} \\
 & -m_n \left\{ [a_{2n}I_1(m_n r) + b_{2n}K_1(m_n r)] \cos(m_n z) \right. \\
 & \quad \left. + \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n r)}{m_n^2} P_n \cos(m_n z) \right\} \quad (44)
 \end{aligned}$$

$$B_{r3} = \sum_{n=1}^{\infty} -m_n [a_{3n}I_1(m_n r) + b_{3n}K_1(m_n r)] \cos(m_n z) \quad (45)$$

$$B_{z1} = \sum_{n=1}^{\infty} m_n [a_{1n}I_0(m_n r) - b_{1n}K_0(m_n r)] \sin(m_n z) \quad (46)$$

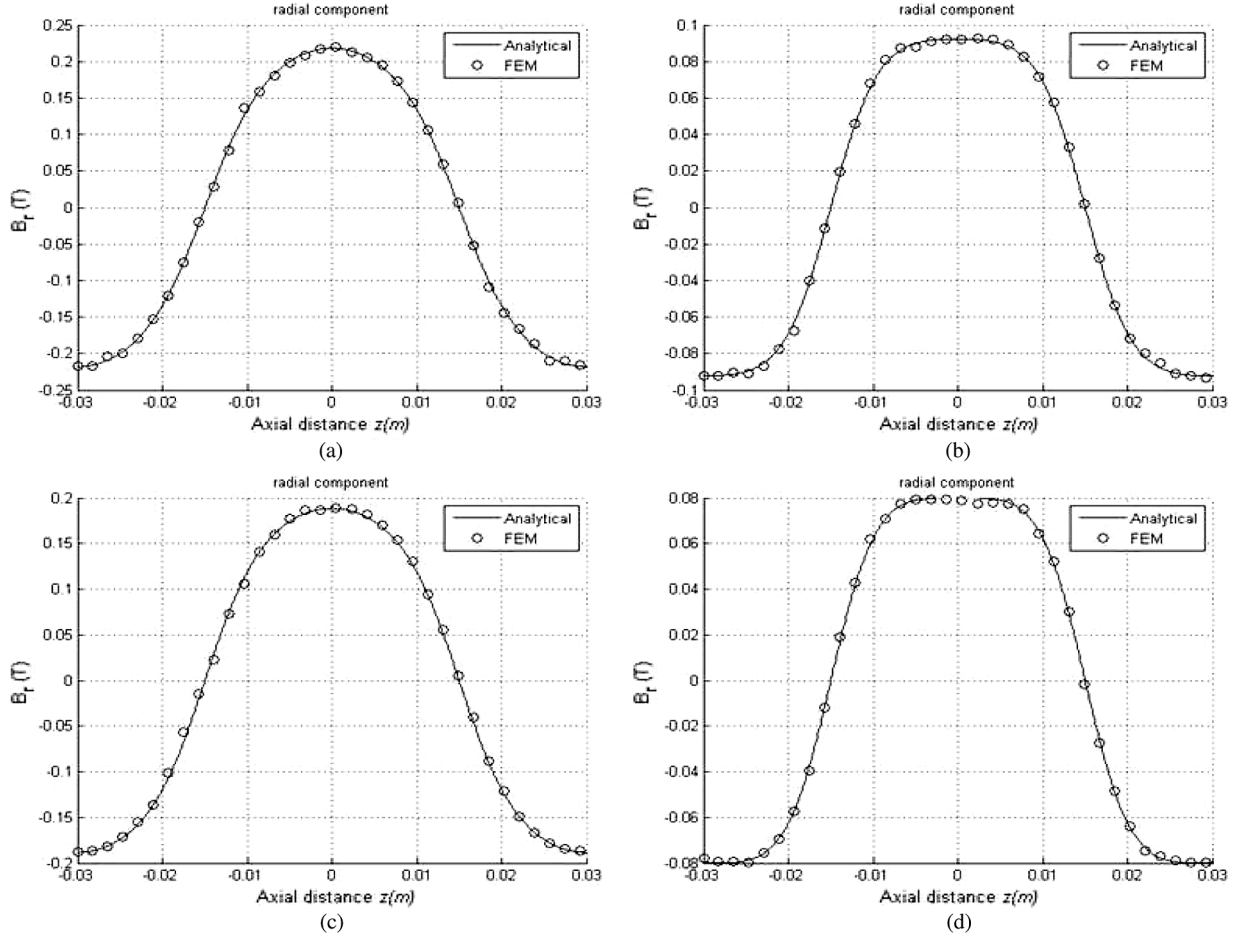


Fig. 7. B_r vs. axial distance z . (a) Double-sided back-irons; (b) external back-iron; (c) internal back-iron; (d) back-iron free.

$$B_{z2} = \sum_{n=1}^{\infty} m_n \left\{ [a_{2n} I_0(m_n r) - b_{2n} K_0(m_n r)] \sin(m_n z) + \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n r)}{m_n^2} P_n \sin(m_n z) \right\} \quad (47)$$

$$B_{z3} = \sum_{n=1}^{\infty} m_n [a_{3n} I_0(m_n r) - b_{3n} K_0(m_n r)] \sin(m_n z). \quad (48)$$

The coefficients are obtained with following equations:

$$\begin{aligned} a_{1n} &= 0, \quad b_{3n} = 0, \\ a_{1n} I_1(m_n R_m) + b_{1n} K_1(m_n R_m) &= a_{2n} I_1(m_n R_m) + b_{2n} K_1(m_n R_m) \\ &+ \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n R_m)}{m_n^2} P_n, \\ \mu_r [a_{1n} I_0(m_n R_m) - b_{1n} K_0(m_n R_m)] &= [a_{2n} I_0(m_n R_m) - b_{2n} K_0(m_n R_m)] \\ &+ \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_m)}{m_n^2} P_n, \\ [a_{2n} I_1(m_n R_r) + b_{2n} K_1(m_n R_r)] &+ \frac{1}{2} \frac{\pi \text{Struve} L_1(m_n R_r)}{m_n^2} P_n \\ &= a_{3n} I_1(m_n R_r) + b_{3n} K_1(m_n R_r), \end{aligned}$$

$$\begin{aligned} \mu_r [a_{3n} I_0(m_n R_r) - b_{3n} K_0(m_n R_r)] &= [a_{2n} I_0(m_n R_r) - b_{2n} K_0(m_n R_r)] \\ &+ \frac{1}{2} \frac{\pi \text{Struve} L_0(m_n R_r)}{m_n^2} P_n. \end{aligned}$$

V. MODEL VALIDATION AND IRON EFFECT ANALYSIS

In this section, the derived flux field models are compared with numerical computations. And the back iron effect on flux density components B_r and B_z are analyzed and compared.

A. Variation of B_r Versus Radial Distance

Fig. 5 illustrates the variation of B_r with respect to radial distance r . The structure parameters used in this computation are $\tau_m = \tau_p = 0.03$ m, $R_r = 0.007$ m, $R_m = 0.015$ m, $R_s = 0.03$ m, $B_{rem} = 1.2$ T, $\mu_r = 1.05$, and $z = \tau_p/4$. For configurations with internal back-iron, the curves start from the boundary between internal back-iron and PM, i.e., $r = 0.007$ m. For configurations without internal back-iron, the curves start from the central line, i.e., $r = 0$. Both analytical models and numerical results are presented in the figures. It shows that the analytical models fit with the numerical results closely. Fig. 5(a) indicates that B_r has the maximum value at the boundary between internal back-iron and PM ($r = R_r$), and then decreases gradually. This is because of the fact that all flux lines in the magnetic loop converge at the boundary. For $r > R_r$, the flux

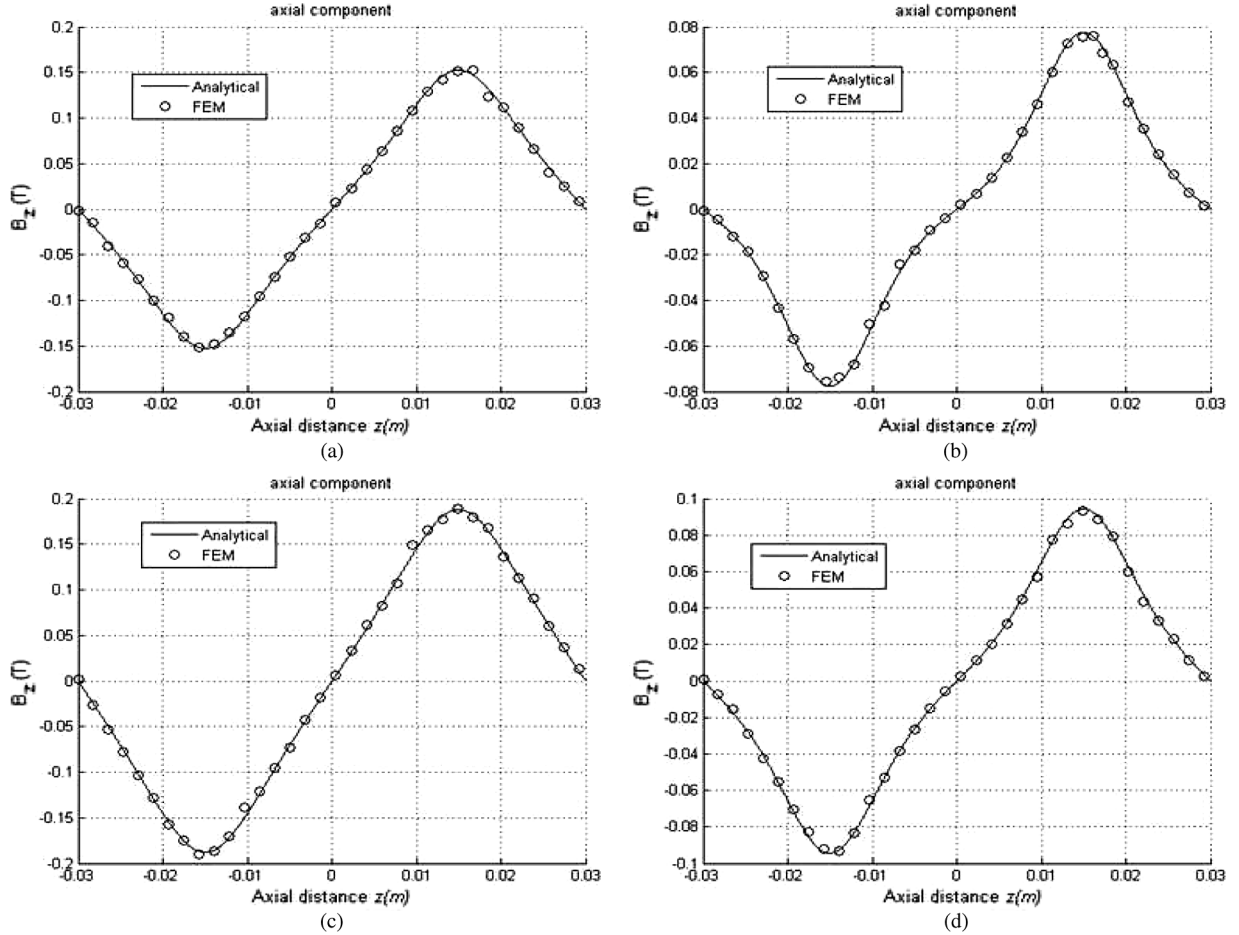


Fig. 8. B_z vs. axial distance z . (a) Double-sided back-irons; (b) external back-iron; (c) internal back-iron; (d) back-iron free.

leakage increases. Fig. 5(b) shows that B_r is equal to zero at $r = 0$ as the flux field is axially symmetric. B_r increases gradually when the point is near to the magnet, arrives the maximum value at the magnet center, and then decreases gradually when r goes to large values. Fig. 5(c) is similar to 5(a), although the latter values are slightly smaller than the former. (For example, in Fig. 5(a), $B_r|_{r=0.03} = 0.12$ T, in Fig. 5(c), $B_r|_{r=0.03} = 0.07$ T). This indicates that the removal of external back-iron may decrease the flux density, but it does not affect too much. Fig. 5(d) is similar to 5(b). However, the flux density values are much smaller than those in 5(a) and 5(c), which indicates that the internal back-iron on the PM enhances the maximum flux density significantly. Thus, employment of internal back-iron is important for the design of PM linear machines.

B. Variation of B_z Versus Radial Distance

Fig. 6 presents the variation of B_z versus radial distance r for four patterns of back-iron arrangements. The variations in Fig. 6(a) and 6(c) are similar. At the boundary between internal back-iron and PM ($r = R_r$), $B_z = 0$ because the flux line is normal to the boundary surface. This also applies to the boundary between external back-iron and coil area in Fig. 6(a) at $r = R_s$. But for the same point at Fig. 6(c), B_z is not equal to zero. B_z jumps at $r = R_m$ (0.015 m) as the tangential components of flux density are not continuous at the boundary. In general, the flux density in Fig. 6(c) is slightly larger than that of

6(a), because the flux leakage is larger in 6(c) due to the removal of external back-iron. The flux variations in Fig. 6(b) and (d) are similar. For Fig. 6(b), $B_z|_{r=0.03} = 0$ due to the back-iron boundary. The removal of external back-iron does not affect the magnetic field significantly.

C. Variation of B_r Versus Axial Distance

Fig. 7 illustrates the variation of B_r with respect to axial distance z . r is fixed at $(R_m + R_s)/2$ for the calculation. The curves are symmetric about $z = 0$ for the four arrangements of back-irons due to the alternative magnetization patterns of PMs in z direction. Furthermore, B_r arrives the maximum value at the center plane of PM ($z = 0$). The top of curves in Fig. 7(b) and (d) is relatively flat, because the external back-iron can “pull” the flux line to r direction, and reduce the flux leakage. The peak values in Fig. 7(a) and (c) are larger than those in Fig. 7(b) and (d). This indicates that the internal back-iron is much more efficient than the external one in terms of the ability to improve the flux density.

D. Variation of B_z Versus Axial Distance

Fig. 8 presents the variation of B_z in the axial direction z for the four back-iron configurations. The variations of flux density in the four figures are similar. B_z is equal to zero at center planes of PM ($z = 0, -0.03$ m and 0.03 m), due to the symmetric arrangement of PMs. Because the flux scatters to both sides from

the center of PM, the direction of flux lines is opposite. Therefore, as indicated in the figures, the signs for two sides of $z = 0$ are opposite.

VI. CONCLUSION

Ferromagnetic materials are widely employed in electromagnetic machines. The objective of this paper is to analyze the effect of back-irons in PM linear machines. The magnetic flux distribution of tubular linear machines with different back-iron patterns is formulated based on magnetic vector potential and Bessel functions. The derived flux field model is validated with FEM calculations. The comparison indicates that analytical model fits with the numerical result closely. Furthermore, the employment of back-iron, especially the internal back-iron, helps to increase the flux density component B_r in the PM linear machines significantly. The component of B_r contributes to the generation of the actuator force output in PM machines. Moreover, B_r is continuous at the boundaries, whereas B_z is not. And B_r is normal to the boundary surfaces with back-irons. This magnetic property can reduce the flux leakages in z direction, and increases the flux component in r direction for force generation.

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Liang Yan (M'07–SM'12) received the Bachelor's degree from North China Institute of Technology, Shanxi, China, in 1995, the Master's degree from Beijing Institute of Technology, Beijing, China, in 1998, and the Ph.D degree from Nanyang Technological University, Singapore, in 2006.

He was with Beijing Institute of Technology from 1998 to 2002, Nanyang Technological University from 2006–2009, and is currently with Beihang University. His research interests include robotics, intelligent actuators & sensors, navigation system and expert system design.

Prof. Yan was the recipient of the National Defense Science and Technology Award, China in 2002. He was the Publication Chairman of 2008 IEEE International Conference on Cybernetics and Intelligent Systems, 2009 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, and 2011 International Conference on Fluid Power and Mechatronics.

Jie Hu received the Bachelor's degree from Beihang University (Beijing University of Aeronautics and Astronautics, BUAA), Beijing, China in 2011. His research interests include electromagnetic actuators, sensors, electrical circuit design.

Mr. Hu was the recipient of the best bachelor thesis of the School of Automation Science and Electrical Engineering, Beihang University, and excellent thesis of BUAA in 2011.

Nan Yao received the Bachelor's and Master's degrees from Beihang University, Beijing, China, in 2003 and 2006, respectively.

She was with Beihang University as a Teaching Assistant from 2006 to 2007, and has been a Lecturer since 2008. Her research interests include robotics, actuators, sensors, distributed interaction simulation, artificial intelligence technology, and network reliability. She has published about 10 papers in international journals and refereed conferences.

Zongxia Jiao received the B.S. and Ph.D. degrees from Zhejiang University, China, in 1985 and 1991, respectively.

He was with Beihang University as a Postdoc from 1991 to 1993, and a Professor since 1994. He was a Visiting Professor at the Institute for Aircraft System Engineering, Technische University Hamburg-Harburg, Germany in 2000. He is currently the College Dean of the School of Automation Science and Electrical Engineering, BUAA. His research interests include actuators, sensors, fluid power, and transmission. He was the recipient of Changjiang Scholar Professor in 2006, and Distinguished Young Scholar in 2008. He served on the 2011 International Conference on Fluid Power and Transmission and 2012 IEEE International Conference on Industrial Informatics as general chair. He has published more than 100 papers in international journals and refereed conferences.

I-Ming Chen (M'95–SM'06–F'12) received the B.S. degree from the National Taiwan University, Taipei, Taiwan, in 1986, and the M.S. and the Ph.D. degrees from the California Institute of Technology, Pasadena, CA, in 1989 and 1994, respectively.

Since 1995, he has been with the School of Mechanical and Aerospace Engineering of Nanyang Technological University (NTU), Singapore. He is the director of Intelligent Systems Center in NTU. He was JSPS Visiting Scholar in Kyoto University, Japan in 1999, Visiting Scholar in the Department of Mechanical Engineering of MIT in 2004, and is currently Fellow of Singapore-MIT Alliance under Manufacturing Systems and Technology (MST) Program. His research interests are in wearable sensors, human-robot interaction, reconfigurable automation, parallel kinematics machines (PKM), and smart material based actuators.

Kok-Meng Lee (M'89–SM'02–F'05) received the B.S. degree from the State University of New York, Buffalo, in 1980, and the S.M. and Ph.D. degrees from the Massachusetts Institute of Technology, Cambridge, in 1982 and 1985, respectively.

Currently, he is a Professor in the Woodruff School of Mechanical Engineering at the Georgia Institute of Technology, Atlanta. His research interests include system dynamics/control, robotics, automation, and mechatronics. He holds eight patents in machine vision, three degrees of freedom (DOF) spherical motor/encoder, and live-bird handling system.

Dr. Lee is a Fellow of ASME. He received the National Science Foundation (NSF) Presidential Young Investigator, Sigma Xi Junior Faculty Research, International Hall of Fame New Technology, and Kayamori Best Paper awards.