

Magnetic Pantographic Exoskeleton Illustrated With a Biomimetic Ankle-Foot Simulator for 3-DOF Noncontact Actuation/Measurements

Zixin Que  and Kok-Meng Lee , *Life Fellow, IEEE*

Abstract—This article presents a 3-DOF planar magnetic pantographic exoskeleton (Mag-PGE) with embedded sensors for manipulating/measuring the motion and force/torque (F/T) of an ankle joint in the sagittal plane. A biomimetic ankle-foot simulator (AFS) is designed to facilitate investigation and help visualize its potential uses for in-bed acute stroke rehabilitation where the affected leg muscles are often passive. Unlike a conventional motor in which the rotor is supported by mechanical bearings on the stator, the (stator and rotor) of the Mag-PGE are separately attached to the (shank and foot) sharing the ankle joint of the human leg such that it enables the foot to flex in the sagittal plane, imposing no mechanical constraints on the ankle joint. A Mag-PGE/AFS model with closed-form solutions to the rotor magnetic field and motor F/T is provided; both forward and inverse problems are considered. The methods to design, train, and calibrate embedded sensors utilizing the rotor magnetic field to measure the rotor/foot motion are numerically and experimentally evaluated. As illustrated with an AFS, the model along with the embedded measurements plays a pivotal role in computing the forward and inverse solutions to the 3-DOF motor force-current model.

Index Terms—Actuator design, embedded sensor design, force and torque models, force-current equation, magnetic sensor, motor.

I. INTRODUCTION

WHEN wearing a robotic exoskeleton on a human body, improper motion restrictions and forced inputs may cause serious injury to the wearer, particularly stroke patients. During the early stage of stroke, the muscles of the affected side remain intact but in a state of low tension, and often lacking force

perception. Inspired by the pantographic exoskeleton (PGE) [1] that traces the joint motion in the presence of misalignments, this article extends the PGE design by providing a direct-drive planar motor [referred to here as a magnetic pantographic exoskeleton (Mag-PGE)] for noncontact manipulating and monitoring the internal motion/force state of an ankle joint. The Mag-PGE is illustrated with a biomimetic ankle-foot simulator (AFS) that will serve as an evaluation platform before its deployment for in-bed acute stroke rehabilitation exercises [2].

Although accurate electric motors are widely available at a reasonably low cost and can be configured as serial elastic actuators by incorporating mechanical springs, they are often designed to operate with a fixed axis. For robot-assisted rehabilitation, the wearable robot forms a closed (human–robot) kinematic chain imposing an externally applied motion/force constraint on the human joint and, thus, must have sufficient DOFs to accommodate natural force/displacement variations within the joint. A mechanism is required to house multiple single-axis permanent-magnet (PM) motors, lead/ball screws, elastic elements, and bearings to constrain the moving components to single-axis rotational/translational motions. These constraining mechanisms are often complex and introduce unwanted friction and additional inertia.

The need for multi-DOF actuators has motivated the development of a ball-joint-like permanent-magnet spherical motor (PMSM) capable of dexterous motion control. Originally proposed in [3] as a robotic wrist actuator characterized by its highly redundant input, several PMSM configurations [4], [5], [6], [7], [8], [9] and control strategies have been developed for applications, including a haptic interface [10], fault remedy [11], robotic joint [12], satellite attitude control [13], and actuators for computer vision [14]. Low-cost, highly compact magnetic flux density (MFD) sensors can be embedded in a PM motor to harness its inherent MFD field to determine the rotor orientation [15] or be directly used for feedback control [16]. More recently, embedded sensors measuring both the MFD field and the back electromotive force (back-emf) of a PMSM have been used with a Kalman filter-based sensor fusion for full state estimation of 3-DOF angular displacement/velocity in real time [17], effectively overcoming sensor noise with improved accuracy relative to the popular inertial measurement unit (IMU) measurements.

Despite the above-mentioned attractive features, the design principle of a multi-DOF PM motor has been underexploited

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Zixin Que is with the State Key Laboratory of Intelligent Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: d202080220@hust.edu.cn).

Kok-Meng Lee is with the Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405 USA (e-mail: kokmeng.lee@me.gatech.edu).

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for stroke rehabilitation. In response to this need, a 3-DOF planar PM motor was designed [18] to accommodate biojoint variations using a distributed current source (DCS) modeling method originally developed in [19]. In [18], the DCS models were verified with exact solutions and commercial finite-element analysis (FEA), showing that the DCS models are accurate within 2.5% of exact solutions and computationally more efficient than FEA as the latter must incorporate large airspace in its solution domain resulting in low efficiency and numerical errors. Built upon the numerical studies [18], this article offers a novel actuation/measurement solution based on a structurally simple electromagnetic system that converts magnetic energy to elastic forces/torque (F/T) for positioning the planar 3-DOF motion through a noncontact magnetic field generated by the rotor PMs and the stator electromagnets (EMs). Because the (stator and rotor) separately attach to the (shank and foot) sharing the ankle joint, they are noncontact; the Mag-PGE requires no bearing/constraining mechanism on the human ankle and is free from unwanted friction (that causes wear/tear) and additional inertias.

The rest of this article is organized as follows.

Section II begins with the design concepts of the Mag-PGE illustrated with a biomimetic AFS and introduces a control-oriented modeling framework for the 3-DOF noncontact planar actuation and measurements. The Mag-PGE and the AFS and their sensing methods are then separately modeled in Section II-A and B, respectively; both the forward/inverse models are considered. To help focus on design innovation and for completeness, the DCS closed-form solutions to the rotor-MFD field capitalized in measuring the ankle-foot motion and computing the force-current equations are derived in the Appendix.

Section III presents the prototype development of a Mag-PGE/AFS and the results organized around a three-folded objective. Based on the physics revealed in the inverse magnetic field model, Section III-A offers a method to train its embedded sensors with a fully connected neural network (FC-NN) and evaluates the Mag-PGE for motion measurements. Section III-B develops a method to calibrate the embedded sensors and the magnitude of the PM magnetization and analyze the FC-NN-based measurements in the presence of sensor noise. With the control-oriented model, Section III-C demonstrates an application illustrated with a prototype Mag-PGE/AFS. Practical issues are highlighted for future studies.

Finally, Section IV concludes this article.

II. DESIGN CONCEPTS AND MODEL FORMULATION

Fig. 1 illustrates the design concept of a 3-DOF Mag-PGE composing a symmetric pair of planar direct-drive motors with embedded MFD sensors for manipulating/measuring the planar motion $[\mathbf{R}_M(X_M, Y_M, 0), \theta]$ and F/T of an ankle joint in the sagittal plane. Fig. 2 shows a biomimetic AFS designed to facilitate investigating the Mag-PGE and its potential uses for in-bed acute stroke rehabilitation where the muscles of the affected leg are often passive.

Mag-PGE: Unlike a conventional motor in which its rotor is supported on the stator by a mechanical bearing, the (stator and

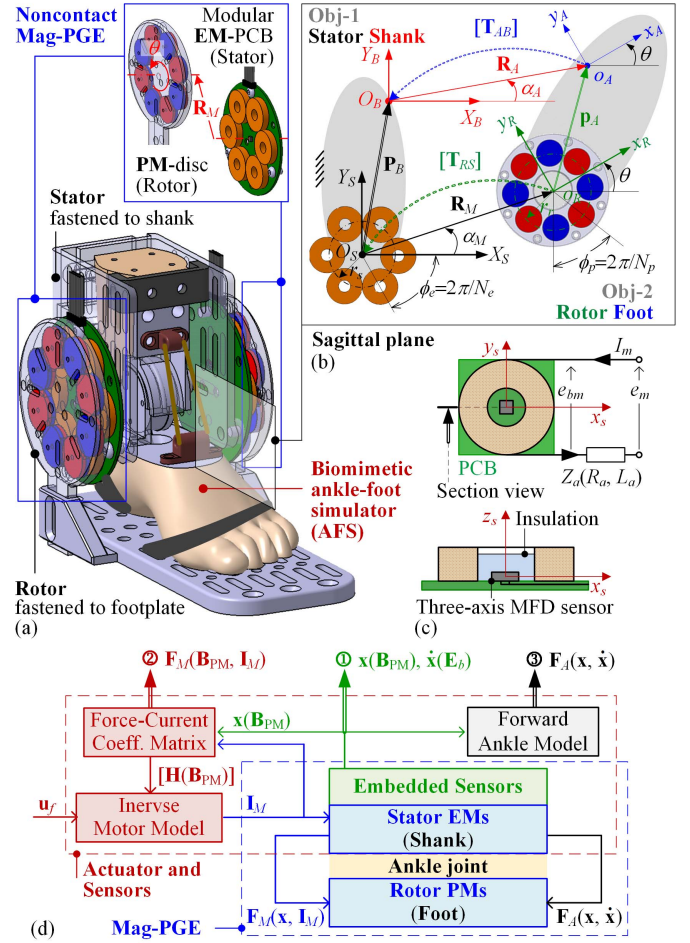


Fig. 1. Design concept of a Mag-PGE/AFS. (a) CAD model. (b) Coordinate systems of the shank-fixed stator and foot-fixed rotor. (c) Modular EM-PCB with embedded MFD sensor. (d) Control-oriented model of a Mag-PGE.

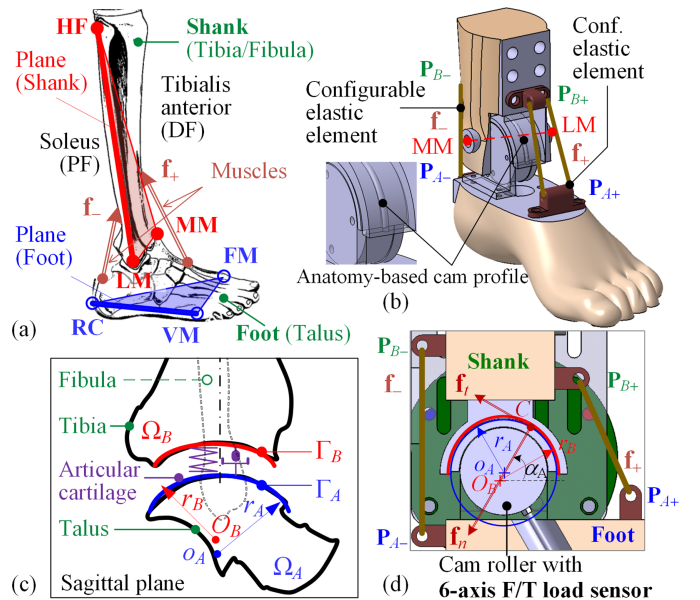


Fig. 2. Biomimetic AFS. (a) Ankle-foot anatomy. (b) CAD model. (c) Ankle-joint parameters. (d) Contact force measurements.

rotor) of the Mag-PGE [see Fig. 1(a)] are separately attached to the (shank and foot) such that the motor shares the ankle joint enabling the foot to flex in the sagittal plane. In Fig. 1(b), the $(X_S Y_S Z_S, x_R y_R z_R)$ are assigned at the centers (O_S, O_R) of the (shank-fixed stator and foot-flexing rotor) and similarly the $(X_B Y_B Z_B, x_A y_A z_A)$ at (O_B, O_A) on the (shank and foot). The rotor/foot is manipulated electromagnetically relative to the stator/shank. The coordinate transformations from (rotor-to-stator and foot-to-shank) are described by the pair of planar 3×3 matrices ($[T_{RS}], [T_{AB}]$).

In Fig. 1(a), (PM and EM) are the acronyms for (permanent magnet and electromagnet) and each modularly designed EM [see Fig. 1(c)] with sensors on the printed circuit board (PCB) measures the MFD field and the voltage e_m across the coil. With no physical contact between the stator and rotor, the Mag-PGE measures the MFD field $\mathbf{B}_{PM}$ of the moving rotor PMs and the back-emf $\mathbf{E}_b = [\dots e_{bm} \dots]^T$ of the stationary EMs, from which the position/orientation $\mathbf{x}(\mathbf{B}_{PM}) = [X_M Y_M \theta]^T$ and velocity of the rotor/foot can be determined from the inverse models. The magnetic F/T vector $\mathbf{F}_M(\mathbf{f}_M, \tau_M)$ due to the current vector $\mathbf{I}_M$ in the $\mathbf{B}_{PM}(\mathbf{x})$ is governed by Lorentz's force equation. Fig. 1(d) illustrates the role $\mathbf{B}_{PM}(\mathbf{x})$ played in the control application, which relates the motion $(\mathbf{x}, \dot{\mathbf{x}})$ due to $\mathbf{F}_M$.

Biomimetic AFS: Fig. 2 shows the biomimetic AFS that houses a commercial six-axis F/T sensor in the cam roller at O_A , where (PF and DF) denote the ankle (plantarflexion and dorsiflexion). As schematically modeled in Fig. 2(a) and (c), where the talus surface Γ_A rolls/slides relative to the tibia surface Γ_B as the foot flexes, the muscle forces $\mathbf{f}_\pm$ exert on the Mag-PGE an impedance load (normal $\mathbf{f}_n$ and tangential $\mathbf{f}_t$) through the compliant contact C . Fig. 2(b) and (d) show the CAD model and parameters/variables used to determine the position/orientation of Γ_A (center O_A) relative to Γ_B (center O_B). With the in-bed acute stroke rehabilitation application in mind, the muscles are assumed passive. Thus, $\mathbf{f}_\pm$ is modeled as a pair of mechanical impedances (stiffnesses $k_\pm$ and damping coefficients $b_\pm$), and the anatomy-based ankle-joint kinematics is simulated using a cam profile Γ_B against which the roller (talus surface Γ_A) rolls and slides. The net forces $\mathbf{F}_A$ contributed by the muscle forces $\mathbf{f}_\pm$ and the contact $(\mathbf{f}_n, \mathbf{f}_t)$ forces act as an impedance on the rotor.

A. Direct-Drive Actuation With Embedded Sensors (See Fig. 1)

As shown in Fig. 1(a), the rotor PMs are evenly distributed on a pair of PM-discs (one on each side of the ankle); similarly, the stator EMs [see Fig. 1(c)] are evenly distributed but on a pair of EM-PCBs. The Mag-PGE model requires the MFD field $\mathbf{B}(\mathbf{x})$ and the force-current coefficient matrix $[\mathbf{H}(\mathbf{x})]$. For readability, the MFD field and magnetic F/T are derived using the DCS method [18], [19] in the Appendix, where each cylindrical PM (radius r_p , length l_p , and magnetization M_o) and each annular EM (outer radius a_o , inner radius a_i , length l_e , and current I_m), as shown in Fig. 3(a), are used in the illustration. The DCS method models a PM as a surface current density (SCD) source and an EM as a volume current density (VCD) source. Since the pairs of planar motors [see Fig. 1(a)] on the left and right of the

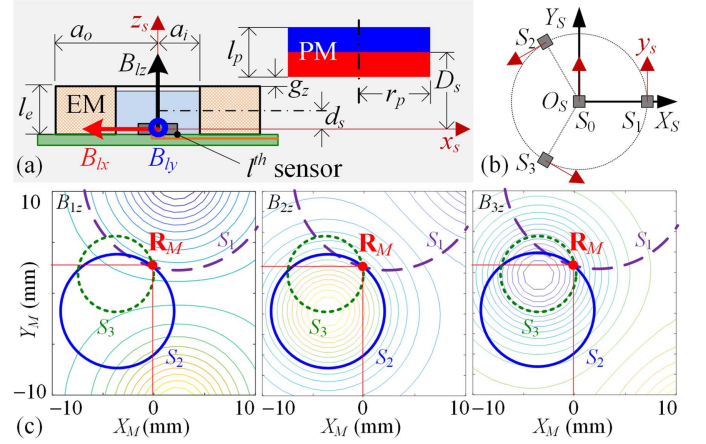


Fig. 3. MFD models for position/orientation sensing. (a) Coordinates and parameters used in the DCS simulation. (b) Sensor placement. (c) Isolines map of $(B_{1z}, B_{2z}, \text{ and } B_{3z})$ for estimating the planar position of the rotor center.

foot are identical and symmetric, they are discriminated by the subscripts $(+, -)$ for simplicity in the mathematical derivations.

1) Magnetic MFD Models for Position/Orientation Sensing: Since the current flowing through the stator EM (height d_s) in which the MFD sensor is embedded on the PCB [see Fig. 3(a)], its EM-generated MFD field is predetermined offline and subtracted from measurements; the MFD measured by the l th sensor at $\mathbf{P}_l$ is referred to here as $\mathbf{B}_l(\mathbf{x}) = \mathbf{B}_{PM}(\mathbf{P}_l)$.

With each PM (height D_s) modeled as an SCD source, the DCS method uniformly discretizes the PM circumferential surface (area S_p) into $K = 1, \dots, N_K$ equal-area elemental SCD sources $\mathbf{K}_{nK}$ centered at $\mathbf{p}_{nK}$ in the rotor coordinate frame O_R . For an axis-symmetric PM, its MFD field at an observed point depends only on the radial distance between the PM and the sensor. With the aid of Fig. 9, where the DCS parameters are defined, the MFD field $\mathbf{B}_l$ contributed by the N_p rotor PMs in a magnetically linear system can be computed from (1a), where μ_0 is the permeability of free space and $n=1, \dots, N_p$

$$\frac{\mathbf{B}_l(\mathbf{x})}{\mu_0 M_o} = \frac{1}{4\pi} \frac{S_p}{N_K} \sum_{n,K} \left[\frac{\mathbf{K}_{nK}}{M_o} \times \left(\frac{\mathbf{L}_{p+}}{L_{p+}^3} + \frac{\mathbf{L}_{p-}}{L_{p-}^3} \right) \right] \quad (1a)$$

$$\text{where } \mathbf{L}_{p\pm}(\mathbf{x}) = \mathbf{P}_l - \mathbf{p}_{nK\pm}, \quad (1b)$$

$$L_{p\pm} = |\mathbf{L}_{p\pm}| \quad (1c)$$

$$\mathbf{p}_{nK\pm}(\mathbf{x}) = [\mathbf{T}_Z(\theta)] \mathbf{p}_{nK\pm} + \mathbf{R}_M \quad (1d)$$

$$[\mathbf{T}_Z(\theta)] = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \quad (1e)$$

$$\mathbf{R}_M = \begin{bmatrix} X_M \\ Y_M \\ Z_M = 0 \end{bmatrix}. \quad (1f)$$

The inverse solution to (1a), which determines the rotor position/orientation $\mathbf{x}(X_M, Y_M, \theta)$ from an MFD measurement, may not be unique but a unique solution can be obtained

by using multiple embedded sensors [see Fig. 1(c)], $\mathbf{B} = [\mathbf{B}_0 \dots \mathbf{B}_l \dots \mathbf{B}_{N_s}]^T$. As an illustration, consider Fig. 3(a) and (b), where $N_s=3$, $S_{1,2,3}$ are in the modular coils evenly spaced on the PCB and S_0 is in the PCB center. Fig. 3(c) simulates the MFD fields for an example (six-EM/eight-PM layout) in which three of its z -axis measurements (B_{1z} , B_{2z} , and B_{3z}) are used to locate the rotor center ($\theta = 0$) in the range $X_M, Y_M \in [-10, 10]$ mm. The eight N52 PMs are cylindrical (radius $r_p = 10$ mm and thickness $l_p = 5$ mm). As shown in Fig. 1(b) and Fig. 3(a), both the (PMs, EMs) are evenly distributed on a circle with radii $r_s = r_r = 30$ mm, and the plane-to-plane distance D_s between the sensor and the PM is 9.15 mm.

The simulated “measured MFD fields” are plotted in Fig. 3(c). The (left, middle, and right) plots are the constant (B_{1z} , B_{2z} , and B_{3z}) contours; on each of these plots, the other two sensors’ contours at a specified center location $\mathbf{R}_M(X_M, Y_M, 0)$ are superimposed. The (S_1 and S_2) contours intercept at two points, and (S_1 and S_3) contours overlap at two points, and the three sensors’ contours intercept at only one point $\mathbf{R}_M$. For $\theta \neq 0$, a set of similar plots can be obtained but the vector $\mathbf{R}_M$ with the same (X_M, Y_M) will intercept at the different constant (B_{1z} , B_{2z} , and B_{3z}) contours, demonstrating that three or more single-axis measurements are needed to ensure a unique (X_M, Y_M, θ) solution.

Because of the symmetric PM layout, the inverse solution repeats every $4\pi/N_p$ as θ increases. As the motion is continuous, a unique θ can be determined based on the smallest change for applications where the measurement sampling period is much smaller than the system time constant. For applications in the presence of sensor noise, an FC-NN is designed in Section III to train the embedded sensors during an offline calibration.

2) Magnetic Force Models for Actuation and F/T Sensing:

For a Mag-PGE with N_e electric current inputs, its output F/T vector $\mathbf{F}_M$ on the rotor/foot is derived in the Appendix where each EM (coil volume V_e) is discretized into $k=1, \dots, N_k$ (equal-volume) elemental VCD sources $\mathbf{J}_{mk}$ centered at $\mathbf{P}_{mk}$ in the stator frame $X_S Y_S Z_S$. From (A.8), where (f_o and τ_o) are defined

$$\mathbf{F}_M (\in \mathbb{R}^{3 \times 1}) = [\mathbf{H}(\mathbf{x})] \mathbf{I}_M (\in \mathbb{R}^{N_e \times 1}) \quad (2a)$$

$$\text{where } [\mathbf{H}(\mathbf{x})] (\in \mathbb{R}^{3 \times N_e}) \\ = \text{diag}(f_o, f_o, \tau_o) [\bar{\mathbf{a}}_1 \dots \bar{\mathbf{a}}_m \dots \bar{\mathbf{a}}_{N_e}]. \quad (2b)$$

For a PM/EM layout/geometry, the column vector $\bar{\mathbf{a}}_m (\in \mathbb{R}^{3 \times 1})$ of the force-current matrix $[\mathbf{H}(\mathbf{x})]$ can be expressed in closed form in (2c), where the angular variables (φ_{mk} , φ_{nK}) of the elemental (VCD, SCD) sources are defined in Fig. 9

$$\bar{\mathbf{a}}_m(\mathbf{x}) = \frac{2}{N_k N_K} \sum_n P_n \sum_{k,K} \frac{\bar{K}}{\bar{L}_{eq}^3} \begin{bmatrix} \cos \varphi_{mk} \\ \sin \varphi_{mk} \\ (r_s/r_p) \sin(\varphi_{mk} - \phi_{em}) \end{bmatrix} \quad (2c)$$

$$\text{where } \bar{K}(\mathbf{x}) = \mathbf{e}_z^T ([\Gamma_Z(\theta)] \mathbf{e}_t(\varphi_{nK}) \times \bar{\mathbf{D}}_{mn}) \quad (2d)$$

$$\text{and } \mathbf{e}_n(*) = [\cos(*) \quad \sin(*) \quad 0]^T, \quad (2e)$$

$$\mathbf{e}_t(*) = d\mathbf{e}_n(*)/d(*). \quad (2f)$$

In (2c), L_{eq} is defined in (A.7c) in terms of the distances $L_{+\pm}$ between the k th VCD and the K th SCD elemental sources from the m th EM to the n th PM pair. In (2d), $\mathbf{e}_z = [0 \ 0 \ 1]^T$.

The Mag-PGE has redundant inputs ($N_e > 3$) to be optimized by solving the inverse model of (2a) for a least-norm solution that minimizes $\|\mathbf{I}_M\|_2$ subject to the constraint imposed by (2a). For a desired force vector $\mathbf{u}_f$, an optimal $\mathbf{I}_M$ can be derived via the Lagrange multiplier leading to the right pseudoinverse (3a) and (b), where $[\mathbf{H}][\mathbf{H}]^T$ is invertible

$$\mathbf{I}_M = [\mathbf{H}]^+ \mathbf{u}_f \quad (3a)$$

$$\text{where } [\mathbf{H}]^+ (\in \mathbb{R}^{N_e \times 3}) = \text{pinv}([\mathbf{H}]) = [\mathbf{H}]^T ([\mathbf{H}][\mathbf{H}]^T)^{-1}. \quad (3b)$$

3) Rotor Velocity Sensing: For a Mag-PGE with N_e EMs [see Fig. 1(a) and (c)] where (R_a and L_a) are the (resistance and self-inductance) of the coil, each with a current I_m flowing through and voltage e_m across it. The power conversion and circuit equation are governed by the conservation of energy (4a) and Kirchhoff’s voltage law (4b)

$$\mathbf{I}_M^T \mathbf{E}_M - (\mathbf{I}_M^T [\mathbf{L}] \dot{\mathbf{I}}_M + \mathbf{I}_M^T R_a \mathbf{I}_M) = \mathbf{F}_M^T \dot{\mathbf{x}}/2 \quad (4a)$$

$$\mathbf{E}_M = \mathbf{E}_b + [\mathbf{L}] \dot{\mathbf{I}}_M + R_a \mathbf{I}_M \quad (4b)$$

where $\mathbf{I}_M, \mathbf{E}_M, \mathbf{E}_b \in \mathbb{R}^{N_e \times 1}$. In (4a) and (4b), the inductance matrix $[\mathbf{L}] \in \mathbb{R}^{N_e \times N_e}$ is symmetric/positive definite, and its (diagonal and off-diagonal) elements are the (self and mutual) inductances among the EMs. For a lossless conversion

$$\mathbf{F}_M = [\mathbf{H}(\mathbf{x})] \mathbf{I}_M \text{ and} \quad (5a)$$

$$\mathbf{E}_b = [\mathbf{H}^T(\mathbf{x})] \dot{\mathbf{x}}/2. \quad (5b)$$

Substituting (5a) into (4a) leads to (4b). Using (4b) along with (5b), the rotor velocity $\dot{\mathbf{x}}$ can be solved from the linear set of equations using the pseudoinverse (pinv) solution

$$\dot{\mathbf{x}} = 2[\mathbf{H}^T(\mathbf{x})]^+ [\mathbf{E}_M - ([\mathbf{L}] \dot{\mathbf{I}}_M + R_a \mathbf{I}_M)]. \quad (6)$$

B. Biomimetic AFS Ankle-Joint Impedance (See Fig. 2)

Fig. 4 defines the parameters/variables of the AFS kinematics/dynamics modeled in Section II-B-1 and B-2.

1) Kinematics and Dynamics of the Rotor/Foot: As shown in Fig. 4(b), Γ_A and Γ_B (with radii r_A and r_B , respectively) roll/slide at the soft-tissue contact point C , where $R_A \approx r_B - r_A$. Unlike the anatomical surfaces (Γ_A, Γ_B) that can be geometrically modeled from magnetic resonance imaging (MRI) images [1], the slide/roll ratio η_{sr} and contact velocity $\mathbf{v}_C$ (relative to O_B) must be determined kinematically. From Fig. 1(b), where $[\mathbf{T}_{AB}][\mathbf{T}_{BS}] = [\mathbf{T}_{AR}][\mathbf{T}_{RS}]$, (R_A, α_A) describing the foot motion can be determined from the measured ($\mathbf{R}_M, \theta$) for known $\mathbf{p}_A$ and $\mathbf{p}_B$ defined in Fig. 1(b)

$$\mathbf{R}_A = \mathbf{R}_M + [\Gamma_Z(\theta)] \mathbf{p}_A - \mathbf{p}_B = [X_A \ Y_A \ Z_A = 0]^T \quad (7a)$$

$$\text{where } R_A = \sqrt{X_A^2 + Y_A^2} \text{ and} \quad (7b)$$

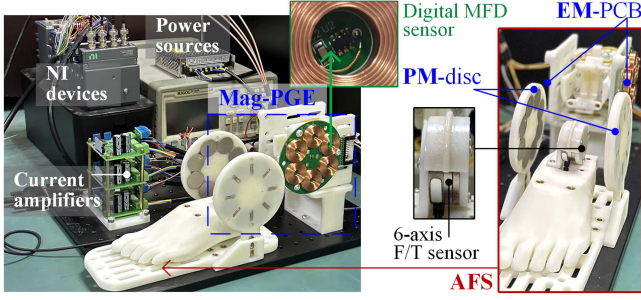


Fig. 5. Prototype Mag-PGE/AFS.

TABLE I
PARAMETERS USED IN THE INVESTIGATION

<i>Mag-PGE geometry (mm):</i> $r_r=32.8$, $r_s=30.0$, $g_z=1.5$, $H=44.1$, $h=51.1$, $D_s=l_p/2+g_z+l_e-0.35$, $d_s=l_e/2-0.35$	
<i>PM</i> ($N_p=8$): $r_p=12.5$, $l_p=5$, $B_o=B_r(l_p/2r_p)=0.29$ T, $B_r(N52)=1.44$ T	
<i>EM</i> ($N_e=6$): $a_o=14.4$, $a_i=5$, $l_e=6$, $d_W=0.35$, $N_W=406$; $R_a=5.3$ Ω , $L_a=2.8$ mH	
<i>Sensors</i> TLE493D-W2B6 A(0/1/2/3)HTSA1, 0.13 mT resolution (12-bit, full range), initial noise $\sigma_s=0.1$ mT	
<i>DCS model</i>	$J_o=7.28$ A/mm ² ; $f_o=B_r J_o V_e=7.12$ N; $\tau_o=r_p f_o=0.09$ Nm
<i>PM (SCD)</i>	$N_K=n_{Ax_i} \times n_{Circ}=5 \times 50$; $s_K=1.57$ mm ²
<i>EM (VCD)</i>	$N_k=n_{Ax_i} \times n_{Rad} \times n_{Circ}=5 \times 5 \times 50$; $v_k=2.72$ mm ³
○ Train dataset ● Validation dataset Training data X_M, Y_M (mm)= [-10: 2: 10]. θ (°)= [-60: 2.50: 30]. Validation data X_M, Y_M (mm)= [-8: 1: 8]. θ (°)= [-50: 1.25: 20].	
<i>Parametric values used in simulation</i> (Dimension in mm)	
Fig. 1 (b): $\mathbf{p}_B=[-2.5 \ -6.50 \ 0]^T$, $\mathbf{p}_A=[-2.5 \ -5.25 \ 0]^T$, $R_A \approx r_B - r_A = 1.25$ mm	
Figs. 2(d) and 4(b): $f_n=10$ N, $\mu=0.2$, $f_t=\mu f_n=2$ N	
Fig. 4(b): $m_c=1.115$ kg, $I_c=4334$ kg $\times$ mm ² , $l_c=46.46$ mm, $\phi_c=-51.47^\circ$	
<i>Elastic stiffnesses:</i> $k_\pm$ experimentally calibrated in Fig. 7(b).	
<i>Trajectory:</i> θ (°)= $35\sin(\omega t + \varphi) - 15$ where $\omega=24^\circ/\text{s}$, $\varphi=\sin^{-1}(3/7)$	
α_A (°)= $\eta\theta + \alpha_{A0}$ where $\eta=0.8$ and $\alpha_{A0}=90^\circ$	
<i>CPU:</i> AMD Ryzen 7 with Radeon Graphics 3.20GHz, 16GB RAM	
<i>Computing:</i> MATLAB R2022a with Parallel Computing Toolbox	
<i>Data acquisition:</i> Arduino Mega2560 <i>Load sensor:</i> ATI Mini40	
<i>XYZ positioner:</i> Resolution: 0.1 μ m <i>Rotation servo:</i> CyberGear	

S_1 , S_2 , and S_3), the DCS models, and the FC-NN are summarized in Table I. As derived in Section II, the rotor-PM magnetic field plays a pivotal role in measuring the rotor position/orientation $\mathbf{x}(X_M, Y_M, \theta)$ and F/T $\mathbf{F}_M(\mathbf{f}_M, \tau_M)$ vectors, which provides the basis for estimating the impedance load $\mathbf{F}_A$ imposed by the ankle joint (AFS) and computing the forward/inverse model [see Fig. 1(d)].

Based on the control-oriented model [see Fig. 1(d)], the results are organized around the three-folded objective. The *first* (see Section III-A) is to design an FC-NN for the embedded sensors to measure the 3-DOF planar motion and numerically evaluate its feasibility. The *second* (see Section III-B) develops an experimental method to calibrate the MFD sensors and identify the PM parameters. The calibrated sensors are then employed to experimentally validate the DCS model of the magnetic field

and evaluate the FC-NN for the embedded sensors. The *third* (see Section III-C) illustrates the forward/inverse force-current models of the Mag-PGE/AFS for a control-oriented application.

A. Design of an FC-NN for the Embedded Sensors

As shown in Fig. 3, three or more single-axis measurements are needed to ensure a unique (X_M, Y_M, θ) solution. The motion measurements are implemented on an FC-NN trained by a backpropagation (BP) learning algorithm in MATLAB using the Levenberg–Marquard function for its convergence speed. The parameters used in this numerical example (based on a six-EM/eight-PM layout) are listed in Table I, where g_z is the air gap between PM and EM [see Fig. 3(a)]. For readability, Section III-A is divided into numerical setup, parametric studies, and discussion of results.

1) *Numerical Setup:* To avoid overfitting and tradeoff between computational accuracy and efficiency, this study investigates the effects of four different input configurations (ICs) using a combination of the four sensors [see Fig. 3(b)] and the network parameters (number of hidden layers and nodes) on the accuracy of the estimated outputs (X_M, Y_M, θ) and the computing times.

- IC1 (3 inputs): Three single-axis sensors: B_{1z} , B_{2z} , B_{3z} .
- IC2 (6 inputs): Three 2-axis sensors: (B_{1x}, B_{1y}) , (B_{2x}, B_{2y}) , (B_{3x}, B_{3y}) .
- IC3 (9 inputs): Three 3-axis sensors: $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$.
- IC4 (12 inputs): Four 3-axis sensors: $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_0$.

Based on the observations of a preliminary study, a larger dataset $[\mathbf{B}_{IC\#}; (X_M, Y_M, \theta)]$ is used for BP training than for validating the FC-NN (see Table I) to account for errors near the boundaries of $(X_M, Y_M) \in [-8, 8]$ mm, 50° plantarflexion (PF) and 20° dorsiflexion (DF). The datasets are computed using the DCS model (1) to serve as ground truth for the investigation. The parameters for training the FC-NN are listed in Table I.

The training performances are evaluated in terms of the root-mean-squared error (RMSE) in (13a). To bound the errors, a maximum absolute error (MAE) is defined in (13b), where $(\bullet) = \Delta \mathbf{X}$, $\Delta \mathbf{Y}$, or $\Delta \theta$, and $\Delta \mathbf{X} = \mathbf{X} - \mathbf{X}_r$, $\Delta \mathbf{Y} = \mathbf{Y} - \mathbf{Y}_r$, and $\Delta \theta = \theta - \theta_r$.

$$\text{RMSE}(\bullet) = \sqrt{(\bullet)^T(\bullet)/N}, \quad (13a)$$

$$\text{MAE}(\bullet) = \max |\bullet|. \quad (13b)$$

2) *Results of the Parametric Studies:* The training/validation results are organized into two groups.

Group 1 (see Table II) identifies the optimal parameters that significantly influence the training errors and computing times, where T_T (s) and T_R (ms) are the time taken to train and validate the FC-NN, respectively. The results of two parametric studies are presented: The *first* validates IC1(3) that represents the minimum number of single-axis MFD inputs to uniquely determine (X_M, Y_M, θ) and analyzes the number of hidden layers/nodes ($HL_{1, 2, 3}$) on the (Ave errors, RMSE) and (T_T , T_R). The *second* compares the effects of the four ICs for a three-hidden-layer (20, 20, 20) FC-NN on the training errors.

TABLE II
NUMERICAL STUDY IDENTIFYING OPTIMAL PARAMETERS

IC1(3)	Training errors (Ave, RMSE)			Time	
HL _{1,2,3}	X_M	Y_M	θ	T_T (s)	T_R (ms)
(15)	-0.09, 4.45	0.00, 4.59	-0.04, 17.82	0.88	6.30
(20)	0.12, 4.36	-0.05, 4.53	0.19, 17.52	1.11	6.53
(20, 15)	0.06, 4.15	-0.03, 4.36	-0.01, 15.81	1.91	9.22
(20, 20)	0.00, 4.08	-0.05, 4.29	0.50, 15.74	2.29	7.68
(25, 20)	-0.04, 4.10	-0.07, 4.33	0.31, 15.86	2.57	7.83
(15, 10, 5)	0.11, 4.24	-0.07, 4.41	0.46, 15.92	1.44	8.02
(20, 15, 10)	0.05, 4.07	-0.03, 4.34	-0.18, 15.74	2.28	8.29
HL(20, 20, 20)					
IC1 (3)	-0.12, 4.04	0.10, 4.26	-0.35, 15.7	4.26	8.63
IC2 (6)	0.00, 0.42	0.00, 0.43	-0.01, 1.60	19.79	8.93
IC3 (9)	0.00, 0.06	0.00, 0.06	0.01, 0.26	35.10	8.94
IC4 (12)	0.00, 0.03	0.00, 0.03	0.00, 0.15	40.55	8.96

TABLE III
EFFECT OF THE SECOND CRITERION ON VALIDATION ERRORS

HL(20, 20, 20)		Validation errors (Ave, RMSE, MAE); T_T (s), T_R (ms)			
Criterion		IC3 (9)		IC4 (12)	
I	X_M	(0.00, 0.05, 0.24)	32.3s	(0.00, 0.03, 0.13)	46.3s
	Y_M	(0.00, 0.05, 0.25)	21.6ms	(0.00, 0.03, 0.11)	23.5ms
	θ	(0.01, 0.19, 1.05)		(0.00, 0.10, 0.50)	
II	X_M	(0.00, 0.04, 0.17)	43.3s	(0.00, 0.03, 0.16)	42.4s
	Y_M	(0.00, 0.04, 0.16)	19.7ms	(0.00, 0.03, 0.12)	20.4ms
	θ	(0.00, 0.12, 0.58)		(0.00, 0.10, 0.49)	
Criterion I		IC4 (12), MAE(ΔX , ΔY , $\Delta \theta$)			
HL ₃		Training		Validation	
10, 10, 10		8.83, 3.05, 26.9		0.77, 0.99, 3.51	
15, 15, 15		0.38, 0.42, 3.22		0.33, 0.34, 1.25	
20, 20, 20		0.40, 0.13, 1.79		0.13, 0.11, 0.50	

Group 2 (see Table III) two different loss functions based on validation errors as a second criterion to minimize the MAE of the IC3(9) and IC4(12)

$$\text{Criterion I: } \sqrt{\text{RMSE}^2(\Delta X) + \text{RMSE}^2(\Delta Y) + \text{RMSE}^2(\Delta \theta)}.$$

$$\text{Criterion II: } \sqrt{\text{MAE}^2(\Delta X) + \text{MAE}^2(\Delta Y) + \text{MAE}^2(\Delta \theta)}.$$

3) Discussions of Results: The findings (see Table II) confirm that IC1(3) with three B_z measurements can uniquely determine the three outputs but has limited accuracy as the training exhibits overfitting when the number of HLs and/or nodes is beyond approximately HL₂(20, 20). Computing accuracy can be greatly improved by increasing the number of inputs with an acceptable increase in training time T_T ; however, its runtime T_R taken to compute the trained FC-NN is relatively constant, varying from 8.63 to 8.96 ms.

Table III demonstrates that the maximum errors can be effectively reduced by imposing a second criterion (I or II) based on the validation errors in the training. As confirmed in Table III in which the errors between the training and validation are compared, the largest errors occur near the boundaries of the training region causing abrupt changes in the estimated values but can be eliminated by training the FC-NN with data from a region larger than the operating range (validation).

B. Sensor Calibration and Its Effects on FC-NN

Fig. 6(a) shows the experimental setup to calibrate the sensors (15b) and (c) and identify the PM parameters (16a) and (b). The

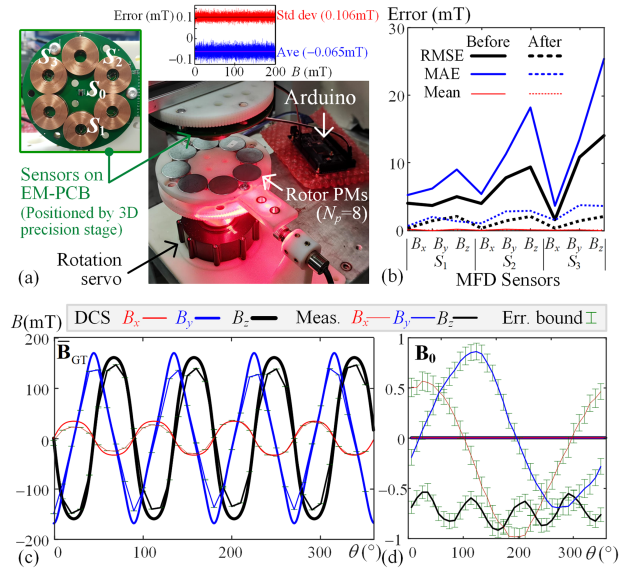


Fig. 6. Sensor calibration. (a) Setup and sensor error. (b) Errors before and after calibration. (c) and (d) DSC model error relative to experimental ($\bar{B}_{GT}$, B_0).

eight-PM rotor and six-EM stator are mounted on a rotation servo and a three-axis precision stage, respectively, which are mechanically aligned with an air gap $g_z=1.5$ mm between them. The sensors are calibrated at ($X_M=Y_M=0$) by measuring the rotor MFD at $N_B=36$ equally spaced points over the full cycle ($0 \leq \theta < 360^\circ$), each with $N_r=40$ repetitions for computing an average. To provide the ground truth for evaluation, the data captured by three sensors (S_1, S_2, S_3) at these 36 points are averaged

$$\bar{B}_{GT} (\in \mathbb{R}^{N_B \times 3}) = [\mathbf{B}_1 + \mathbf{B}_2(-120^\circ) + \mathbf{B}_3(-240^\circ)]/3$$

$$\text{where } \mathbf{B}_l (\in \mathbb{R}^{N_B \times 3}) = [\cdots \mathbf{B}_{ls} \cdots]^T \quad (14)$$

where $\mathbf{B}_{ls} \in \mathbb{R}^{3 \times 1}$ ($l=1, 2, 3; s=1, \dots, N_B$) are experimentally measured at N_B observed points.

For sensor calibration and PM parameter identification, the MFD measurements are expressed in terms of the coefficients, $\beta_l = [\beta_{l0}; \beta_{lX}; \beta_{lY}; \beta_{lZ}]_{4 \times 3}$ and (β_0, β_r) . Considering all N_B measurements, (15a) is formulated to solve for β_l in (15c) from which the calibrated S_l sensor output is given by (15d)

$$\bar{B}_{GT} = [\mathbf{B}_l] \beta_l, \quad (15a)$$

$$\text{where } [\mathbf{B}_l] = [1_{N_B \times 1} \quad \mathbf{B}_l]. \quad (15b)$$

Sensor calibration equation:

$$\beta_l = ([\mathbf{B}_l]^T [\mathbf{B}_l])^{-1} [\mathbf{B}_l]^T \bar{B}_{GT} \quad (15c)$$

$$\hat{B}_l = [\mathbf{B}_l] \beta_l. \quad (15d)$$

The identification of the PM parameters (β_0, β_r) is formulated similarly in (16a), but unlike (15c) calibrating each sensor, it

TABLE IV
PARAMETERS AND EXPERIMENTAL RESULTS

$[\beta_1]=$	$[\beta_2]=$	$[\beta_3]=$
$\begin{bmatrix} 0.125 & -0.115 & -0.053 \\ 0.988 & 0.016 & -0.109 \\ -0.004 & 0.982 & 0.049 \\ 0.033 & -0.024 & 0.968 \end{bmatrix}$	$\begin{bmatrix} -0.089 & 0.130 & 0.186 \\ 1.001 & 0.051 & 0.126 \\ -0.013 & 1.012 & 0.099 \\ -0.034 & -0.061 & 1.039 \end{bmatrix}$	$\begin{bmatrix} -0.062 & -0.026 & -0.130 \\ 0.988 & -0.066 & 0.012 \\ 0.015 & 0.986 & -0.141 \\ -0.002 & 0.079 & 0.971 \end{bmatrix}$
$B_r _{\text{Assumed}}=1.44 \text{ T}, \beta_r=0.8743, B_r _{\text{Measured}}=1.26 \text{ T}$		
Errors (Ave, RMSE); Max $ B_j $ of DCS models		
B_x (mT) (-1.13, 4.67); 33.83	B_y (mT) (-0.01, 16.58); 170.1	B_z (mT) (-0.62, 17.63); 160.9
HL(20, 20, 20)	Errors (Ave, RMSE, MAE), T_T (s) or T_R (ms)	
	Training	Validation
IC3(9)	(0.04°, 0.06°, 0.14°), 0.77	(0.04°, 0.06°, 0.14°), 9.17
IC4(12)	(0.03°, 0.07°, 0.26°), 0.82	(0.03°, 0.07°, 0.26°), 8.82

finds a common factor (16b) for all the rotor PMs

$$\begin{bmatrix} \tilde{\mathbf{B}}_{GT,X} \\ \tilde{\mathbf{B}}_{GT,Y} \\ \tilde{\mathbf{B}}_{GT,Z} \end{bmatrix}_{3N_B \times 1} = \begin{bmatrix} 1 & \tilde{\mathbf{B}}_X \\ 1 & \tilde{\mathbf{B}}_Y \\ 1 & \tilde{\mathbf{B}}_Z \end{bmatrix}_{3N_B \times 2} \begin{bmatrix} \beta_0 \\ \beta_r \end{bmatrix}, \quad (16a)$$

$$\beta_r = \frac{B_r|_{\text{Measured}}}{B_r|_{\text{Assumed}}}. \quad (16b)$$

In (16a), $\tilde{\mathbf{B}}$ is the manufacturer's value used in the derivation of the DCS models.

The results are presented in Table IV and Fig. 6(a)–(c), from which some observations along with an error analysis are made. Fig. 6(b) compares sensor errors before and after calibration to demonstrate the effectiveness of the calibration. As shown in Table IV where the (Ave errors, RMSE) component errors and their Max $|\beta_j|$ values, where $j = x, y, z$ are tabulated, the DCS models closely predict the rotor-MFD field [see Fig. 6(c)] with B_x having the largest error (3% Ave errors, 15% RMSE) of its max B_x value 33.83 mT. The (Ave errors, RMSE) of B_y and B_z are (0, 10)% of their respective max values.

To gain insights into the calibration effects on the FC-NN, the fourth sensor S_0 is left noncalibrated [with errors shown in Fig. 6(d)] so that its influences on the estimated θ can be quantified by comparing with IC3 and IC4; both were trained and validated using the same dataset. To avoid repetition, only $\theta \in [0, 90^\circ]$ is analyzed as its accurate estimation is significantly more difficult than the positions (see Table II). In contrast to Table II, where IC4 has the best performance when trained/validated with noise-free data, IC3 (nine calibrated inputs) displays better performance than IC4 (with three noncalibrated inputs) in the presence of sensor noise (see Table IV, last two rows).

Based on the manufacturer's data, the effects of sensor noise ($\sigma_s = 0.1$ mT) and resolution (0.13 mT) on measurements are simulated to determine the error bound [see Fig. 6(a)] for analyzing the errors observed in Fig. 6(c) and (d), where the error bounds are superimposed on the (noncalibrated $\mathbf{B}_0$, calibrated $\tilde{\mathbf{B}}_{GT}$) to help visualize the error analysis.

1) $\tilde{\mathbf{B}}_{GT}$ has a magnitude two-order higher than $\mathbf{B}_0$.

2) Because of the sensor noise, the overlapping $\mathbf{B}_0$ components destroy uniqueness and degrade IC4 performance.

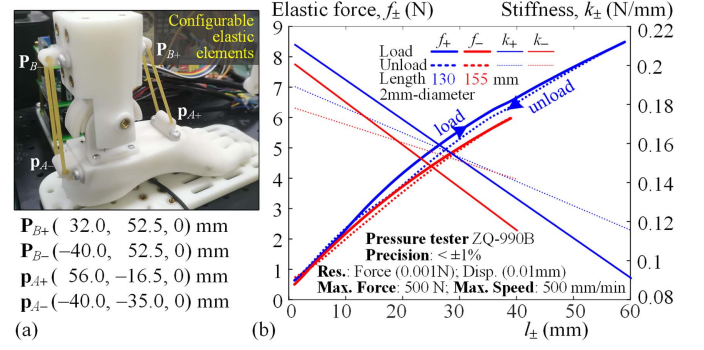


Fig. 7. Prototype AFS. (a) Elastic elements. (b) Experimentally obtained mechanical properties of two different elastic elements.

3) Theoretically, $\mathbf{B}_0 = \mathbf{0}$ at $X_M = Y_M = 0$; $\mathbf{B}_0 \neq \mathbf{0}$ [see Fig. 6(d)] suggests that the stator and rotor planes are not perfectly parallel, resulting in the discrepancy between the DCS model and the measurements.

C. Illustrative Example (With AFS Behaves as Load)

Fig. 7(a) and (b) shows the prototype AFS and the mechanical properties of two different elastic elements experimentally obtained for simulating the nonlinear effects of human muscles behaving as impedance loads for control applications where the solutions to motor force-current model [see Fig. 1(d)] are required. The control-oriented model is simulated numerically (with the parameters and (θ, α_A) trajectories specified in Table I) in Fig. 8, where the (first, second, and third) rows simulate the (kinematics, joint forces, and force-current solutions) of a typical (PF and DF) cycle. The first row (left) shows the (θ, α_A) trajectories with which the (o_A, o_R) motions are simulated in the (middle, right) figures using (7a)–(c). In the second row (right), $\mathbf{F}_A = [f_{AX} f_{AY} \tau_{AZ}]^T$ can then be computed from (11a) for a given set of experimentally simulated [muscle $f_{\pm}$, contact $(\mathbf{f}_n, \mathbf{f}_t)$] forces in the [left, middle] figures. The (left, right) plots of the last row simulate the required $\mathbf{F}_M = [f_{MX} f_{MY} \tau_{MZ}]^T$ based on the dynamics (10) and the corresponding optimal $\mathbf{I}_m$ vector to generate $\mathbf{u}_f$ (desired $\mathbf{F}_M$) using the inverse model (3a and b).

IV. CONCLUSION

The design concept of a novel Mag-PGE with embedded sensors, along with a control-oriented model, has been illustrated with an AFS. The Mag-PGE design can be regarded as a new school of thought: it composes a direct-drive planar motor in which its stator and rotor are noncontact by sharing the ankle joint, imposing no mechanical constraints on the human leg and, thus, free from unwanted friction, additional inertias, and externally applied reactions commonly associated with conventional multiaxis motors relying on mechanical bearings to support the moving components. The model shows that the rotor-MFD field plays a pivoted role in measuring the ankle-foot motion and computing the forward/inverse force-current models essential for feedback control of the ankle-foot system. Motivated by this observation, an FC-NN has been designed for the embedded

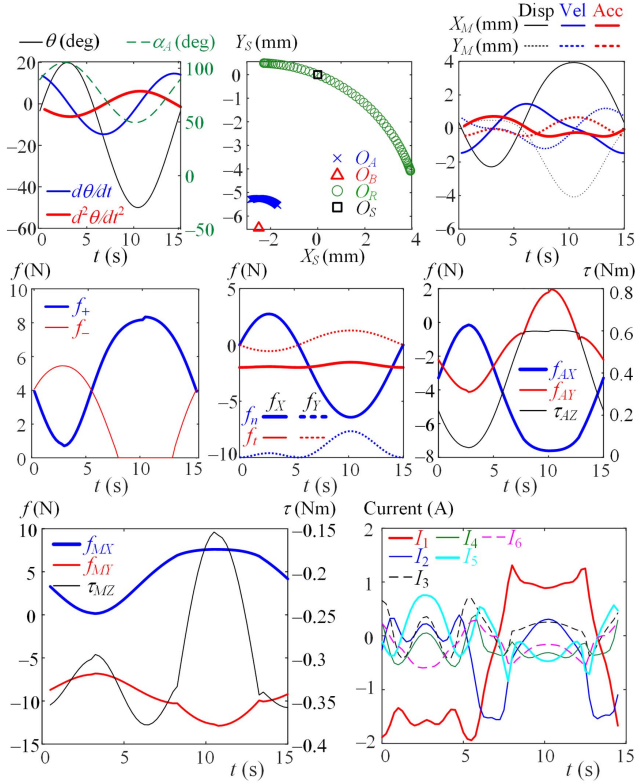


Fig. 8. Results (left to right). (Top): θ -trajectory, coordinate origins, rotor trajectory. (Middle): Elastic, contact, and ankle-joint force. (Bottom): Force (forward model) and current (inverse model).

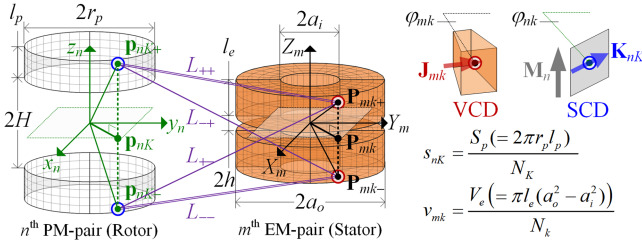


Fig. 9. Schematics and parameters illustrating the DCS methods.

sensors utilizing the rotor-MFD field to measure the rotor/foot motion; numerical results confirm the feasibility of measuring the planar position/orientation with three single-axis MFD sensors and the accuracy can be improved by increasing the number of sensor inputs. A prototype Mag-PGE/AFS has been developed along with the development of an experimental method to calibrate the MFD sensors, with which the DCS-modeled MFD field and the FC-NN are experimentally validated. The current design assumes that the left and right pair of the motor is symmetric, which can be relaxed to provide out-of-plane motion in future studies.

APPENDIX

DCS MODELS [18], [19]

Fig. 9 defines the parameters for modeling the forces acting on the m th current-carrying EM pair (VCD $\mathbf{J}_m$) in the MFD field of the n th PM pair (SCD $\mathbf{K}_n$), where the subscripts (+, -) indicates the (left, right) motors [see Fig. 1(a)]; and the (m th EM $_{\pm}$ and n th PM $_{\pm}$) are symmetric about the (XY and xy) planes. The (SCD and VCD) are uniformly discretized into (N_K equal area and N_k equal volume) elemental (SCD and VCD) sources

$\bar{\mathbf{K}}_{nK} = \frac{\mathbf{K}_{nK}}{P_n M_o} = [\Gamma_Z(\theta)] \mathbf{e}_t(\varphi_{nK})$, where $P_n = (-1)^{n-1}$

$$\bar{\mathbf{J}}_{mk} = \frac{\mathbf{J}_{mk}}{I_m J_o} = \mathbf{e}_t(\varphi_{mk}), \text{ where } J_o = \frac{I_o N_W}{(a_o - a_i) l_e}. \quad (\text{A.2})$$

In (A.2), the ratio I_m ($I_o = 1$ A) is a dimensionless value of the m th EM's current. The EM is discretized uniformly in circumferential and axial direction but ($a_o - a_i$) is divided into n_{Rad} layers radially such that the j th elemental VCD source between two adjacent edges is described by

$$2a_j = \sqrt{a_i^2 + (j-1)(a_o^2 - a_i^2)/n_{\text{Rad}}} + \sqrt{a_i^2 + j(a_o^2 - a_i^2)/n_{\text{Rad}}}.$$

In Fig. 9, $\mathbf{P}_{nK\pm}$ and $\mathbf{P}_{mk\pm}$ form a trapezoid symmetric about the XY -plane implying $L_{\pm\pm} = L_{\mp\mp}$ and $\mathbf{L}_{\pm\pm} + \mathbf{L}_{\mp\mp} = 2(\mathbf{P}_{mk} - \mathbf{P}_{nK})$. To facilitate design analysis, the position, MFD, force, and torque vectors are normalized according to (A.3a–d)

$$\frac{\mathbf{P}}{r_p \bar{\mathbf{P}}} = \mathbf{I}_{3 \times 3} \quad (\text{A.3a})$$

$$\frac{\mathbf{B}}{B_o \bar{\mathbf{B}}} = \mathbf{I}_{3 \times 3} \quad (\text{A.3b})$$

$$\frac{\mathbf{f}}{f_o \bar{\mathbf{f}}} = \mathbf{I}_{3 \times 3} \quad (\text{A.3c})$$

$$\frac{\boldsymbol{\tau}}{\tau_o \bar{\boldsymbol{\tau}}} = \mathbf{I}_{3 \times 3} \quad (\text{A.3d})$$

$$\text{where } B_o = B_r(l_p/2r_p), \quad (\text{A.3e})$$

$$B_r = \mu_0 M_o, \quad (\text{A.3f})$$

$$f_o = B_o J_o V_e, \quad (\text{A.3g})$$

$$\text{and } \tau_o = r_p f_o. \quad (\text{A.3h})$$

Given the normalized MFD field of the rotor PMs in (A.4a–e), the F/T on the stator EMs can then be computed from (A.5a and b) and (A.6) by summing the individual forces $\bar{\mathbf{f}}_{mk\pm}$ between the SCD elements $\bar{\mathbf{P}}_{nK\pm}$ of the stator PMs and the VCD elements $\bar{\mathbf{P}}_{mk\pm}$ of the rotor EMs

$$\bar{\mathbf{B}}_{nK} (\bar{\mathbf{P}}_{mk\pm}) = P_n \left[\frac{\bar{\mathbf{K}}_{nK}}{N_K} \times \left(\frac{\bar{\mathbf{L}}_{\pm\pm}}{\bar{L}_{\pm\pm}^3} + \frac{\bar{\mathbf{L}}_{\pm-}}{\bar{L}_{\pm-}^3} \right) \right] \quad (\text{A.4a})$$

$$\text{where } \bar{\mathbf{L}}_{\pm\pm} = \bar{\mathbf{P}}_{mk\pm} - \bar{\mathbf{P}}_{nK\pm}, \quad (\text{A.4b})$$

$$\bar{\mathbf{L}}_{\pm-} = \bar{\mathbf{P}}_{mk\pm} - \bar{\mathbf{P}}_{nK-}, \quad (\text{A.4c})$$

$$\bar{L}_* = |\bar{\mathbf{L}}_*| \quad (\text{A.4d})$$

$$\text{and } \bar{\mathbf{P}}_{nK\pm} = [\Gamma_Z(\theta)] \bar{\mathbf{P}}_{nK\pm} + \bar{\mathbf{R}}_M \quad (\text{A.4e})$$

$$\bar{\mathbf{f}}_M = \sum_{m=1}^{N_e} \sum_{k=1}^{N_k} [\bar{\mathbf{f}}_{mk+} + \bar{\mathbf{f}}_{mk-}] \quad (\text{A.5a})$$

$$\text{where } \bar{\mathbf{f}}_{mk\pm} = I_m \left[\frac{\bar{\mathbf{J}}_{mk}}{N_k} \times \sum_{n=1}^{N_p} \sum_{K=1}^{N_K} \bar{\mathbf{B}}_{nK} (\bar{\mathbf{P}}_{mk\pm}) \right] \quad (\text{A.5b})$$

$$\bar{\boldsymbol{\tau}}_M = \sum_{m=1}^{N_e} \sum_{k=1}^{N_k} [\bar{\mathbf{P}}_{mk+} \times \bar{\mathbf{f}}_{mk+} + \bar{\mathbf{P}}_{mk-} \times \bar{\mathbf{f}}_{mk-}]. \quad (\text{A.6})$$

Using (A.4a–e), the elemental F/T vectors reduce to (A.7a and b)

$$\frac{\bar{\mathbf{f}}_{mn}}{I_m P_n} = \sum_{k,K} \left(\frac{2}{N_k N_K \bar{L}_{eq}^3} e_t(\varphi_{mk}) \times [\mathbf{\Gamma}_Z(\theta)] e_t(\varphi_{nK}) \times \bar{\mathbf{D}}_{mn} \right) = \begin{bmatrix} \bar{f}_{mn,X} \\ \bar{f}_{mn,Y} \\ 0 \end{bmatrix} \quad (\text{A.7a})$$

$$\frac{\bar{\boldsymbol{\tau}}_{mn}}{I_m P_n} = [\mathbf{\Gamma}_Z(\theta) \bar{\mathbf{p}}_m] \times \bar{\mathbf{f}}_{mn} = [0 \quad 0 \quad \bar{\tau}_{mn,Z}]^T \quad (\text{A.7b})$$

$$\text{where } \bar{L}_{eq}^{-3}(\mathbf{x}) = \bar{L}_{++}^{-3} + \bar{L}_{+-}^{-3} \quad (\text{A.7c})$$

$$\text{and } \bar{\mathbf{D}}_{mn}(\mathbf{x}) = \bar{\mathbf{P}}_{mk} - ([\mathbf{\Gamma}_Z(\theta)] \bar{\mathbf{p}}_{nK} + \bar{\mathbf{R}}_M). \quad (\text{A.7d})$$

The normalized force has the form (A.8)

$$\bar{\mathbf{F}}_M = \bar{\mathbf{f}}_M + \bar{\boldsymbol{\tau}}_M = [\bar{\mathbf{a}}_1 \dots \bar{\mathbf{a}}_m \dots \bar{\mathbf{a}}_{N_e}] \mathbf{I}_M. \quad (\text{A.8})$$

In (A.8), column vectors $\mathbf{I}_M = [\dots I_m \dots]^T$ and $\bar{\mathbf{a}}_m$ can be derived by summing the elemental F/T (A.7a and b) due to the MFD field of the N_p PMs; a closed-form expression is given in Section III-A-2.

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